## CBSEXID2025

## Chapter and Topic-Wise <br> Solved Papers 2011-2024

(All Sets : Delhi \& All India)

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## $>$ PREFACE

Physics is a tricky subject. Your basic concepts of Physics need to be in place if you want to excel in Board Examination. At Career Launcher, our goal is not only to help you maximize your scores in Class XII Physics Board Exam, but also to lay a strong foundation in the subject to help you get ahead in your college and professional career. Over the last decade, we all have seen how the question paper pattern of Class XII Physics paper has kept changing. Bearing in mind this unpredictable nature of Class XII board papers, we've come up with Chapter-wise Solved Papers for Physics for Class XII - to help you prepare better and face the Boards with confidence.
Exclusively designed for the students of CBSE Class XII by highly experienced teachers, the book provides solution to all actual questions of Physics Board Exams conducted from 2011 to 2024. The solutions have been prepared exactly in coherence with the latest marking pattern; after a careful evaluation of previous year trends of the questions asked in Class XII Boards and actual solutions provided by CBSE.
The book follows a three-pronged approach to make your study more focused. The questions are arranged Chapter-wise so that you can begin your preparation with the areas that demand more attention. These are further segmented topic-wise and eventually the break-down is as per the marking pattern. This division will equip you with the ability to gauge which questions require more emphasis and answer accordingly. Apart from this, several value-based questions have also been included.
We hope the book provides the right exposure to Class XII students so that you not only ace your Boards but mold a better future for yourself. And as always, Career Launcher's school team is behind you with its experienced gurus to help your career take wings.
Let's face the Boards with more confidence!
Wishing you all the best,

## Team CL

## Blueprint \& Marks Distribution

Class $12^{\text {th }}$ Physics 2024-25 Analysis Unit Wise
Time : 3 hrs.
Max Marks : 70

|  |  | No. of Periods | Marks |
| :---: | :---: | :---: | :---: |
| Unit-I | Electrostatics | 26 | 16 |
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| Unit-IX | Electronic Devices | 10 | 7 |
|  | Chapter-14: Semiconductor Electronics: Materials, Devices and Simple Circuits |  |  |
|  | Total | 160 | 70 |

Chapter-1: Electric Charges and Fields
Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution. Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.
Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

## Chapter-2: Electrostatic Potential and Capacitance

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.
Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarisation, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

## UNIT II: CURRENT ELECTRICITY

18 Periods

## Chapter-3: Current Electricity

Electric current, flow of electric charges in a metallic conductor, drift velocity, mobility and their relation with electric current; Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical energy and power, electrical resistivity and conductivity, Carbon resistors, colour code for carbon resistors; series and parallel combinations of resistors; temperature dependence of resistance.
Internal resistance of a cell, potential difference and emf of a cell, combination of cells in series and in parallel, Kirchhoff's laws and simple applications, Wheatstone bridge.

UNIT III: MAGNETIC EFFECTS OF CURRENT AND MAGNETISM
25 Periods

## Chapter-4: Moving Charges and Magnetism

Concept of magnetic field, Oersted's experiment.
Biot - Savart law and its application to current carrying circular loop.
Ampere's law and its applications to infinitely long straight wire. Straight and toroidal solenoids (only qualitative treatment), force on a moving charge in uniform magnetic and electric fields.
Force on a current-carrying conductor in a uniform magnetic field, force between two parallel current-carrying conductors-definition of ampere, torque experienced by a current
loop in uniform magnetic field; moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter.
Chapter-5: Magnetism and Matter
Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis, torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; earth's magnetic field and magnetic elements.
Para-, dia- and ferro-magnetic substances, with examples. Electromagnets and factors affecting their strengths, permanent magnets.

## UNIT IV: ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

24 Periods

## Chapter-6: Electromagnetic Induction

Electromagnetic induction; Faraday's laws, induced EMF and current; Lenz's Law, Eddy currents. Self and mutual induction.

## Chapter-7: Alternating Current

Alternating currents, peak and RMS value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, power factor, wattless current.
AC generator and transformer.

## UNIT V: ELECTROMAGNETIC WAVES

## 4 Periods

## Chapter-8: Electromagnetic Waves

Basic idea of displacement current, Electromagnetic waves, their characteristics, their Transverse nature (qualitative ideas only).
Electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays) including elementary facts about their uses.

## UNIT VI: OPTICS

30 Periods

## Chapter-9: Ray Optics and Optical Instruments

Ray Optics: Reflection of light, spherical mirrors, mirror formula, refraction of light, total internal reflection and its applications, optical fibres, refraction at spherical surfaces, lenses, thin lens formula, lensmaker's formula, magnification, power of a lens, combination of thin lenses in contact, refraction and dispersion of light through a prism.

Optical Instruments: Microscopes and astronomical telescopes (reflecting and refracting) and their magnifying powers.

## Chapter-10: Wave Optics

Wave Optics: Wave front and Huygen's principle, reflection and refraction of plane wave at a plane surface using wave fronts. Proof of laws of reflection and refraction using Huygen's principle. Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light, diffraction due to a single slit, width of central maximum.

UNIT VII: DUAL NATURE OF RADIATION AND MATTER
8 Periods

## Chapter-11: Dual Nature of Radiation and Matter

Dual nature of radiation, Photoelectric effect, Hertz and Lenard's observations; Einstein's photoelectric equation-particle nature of light.
Matter waves-wave nature of particles, de-Broglie relation.

## UNIT VIII: ATOMS AND NUCLEI

## Chapter-12: Atoms

Alpha-particle scattering experiment; Rutherford's model of atom; Bohr model of hydrogen atom, Expression for radius of nth possible orbit, velocity and energy of electron in nth orbit, hydrogen line spectra (qualitative treatment only).
Chapter-13: Nuclei
Composition and size of nucleus, nuclear force
Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number; nuclear fission, nuclear fusion.

## UNIT IX: ELECTRONIC DEVICES

10 Periods

## Chapter-14: Semiconductor Electronics: Materials, Devices and Simple Circuits

Energy bands in conductors, semiconductors and insulators (qualitative ideas only) Intrinsic and extrinsic semiconductors- $p$ and $n$ type, $p$ - j junction
Semiconductor diode - I-V characteristics in forward and reverse bias, application of junction diode-diode as a rectifier.

## Electric Charges and Fields

## [Topic 1] Coulomb's law, electrostatic field and electric dipole

## Summary

## Electric Charge

- Electrostatic charge is a fundamental property of matter due to which it produces and experiences electrical and magnetic effects.
- Properties of atoms, molecules and bulk matter are determined by electric and magnetic forces.
- It can be inferred from simple experiments based on frictional electricity that there are two type of charges in nature: negative and positive; and like charges repel and unlike charges attract.
- By convention, the charge on electron is considered as negative and the charge on proton is considered as positive and the charge present is equal. The S.I. unit of electric charge is coulomb. Its C.G.S unit is stat coulomb.
- The nature and amount of electric charge present in a charged body is detected by Gold-leaf electroscope.
- Total charge on a body is expressed as $q= \pm$ ne.


## Conductors and Insulators

- Objects that allow charges to flow through them are called Conductors (metals) and objects that do not allow charges to flow through are called Insulators (rubber, wood, and plastic).
- Objects that behave as an intermediate between conductors and insulators are called semiconductors, for example- silicon.
- The process of sharing charges with the earth, when we bring a charged body in contact with the earth is called grounding or earthing.


## Charging by Induction

- Charging by induction means charging without contact.
- If a plastic comb is rubbed with wool, it becomes negatively charged.


## Three basic properties of electric charge

- Quantization: When the total charge of a body is an integral multiple of a basic quantum of charge, this is known as quantization of electric charge. i.e., $q=$ ne where
$\mathrm{n}= \pm 1, \pm 2, \pm 3$, $\qquad$
- Additivity: It means that the total charge of a system is the algebraic sum (adding taking into account negative and positive signs both) of all the charges in the system.
- Conservation of charge: Conservation of electric charges means that there will be no change in the total charge of the isolated system with time. There is transfer of the electric charge from one body to another, but no charge will be created or destroyed.


## Coulomb's law

The force between two point charges $q_{1}$ and $q_{2}$ is directly proportional to the product of the two charges $\left(q_{1} q_{2}\right)$ and inversely proportional to the square of the distance between them $\left(\mathrm{r}^{2}\right)$ and it acts along the straight line joining the two charges.
$\mathrm{F}_{12}=$ force on $\mathrm{q}_{2}$ due to $\mathrm{q}_{1}=\frac{\mathrm{k}\left(\mathrm{q}_{1} \mathrm{q}_{2}\right)}{\mathrm{r}^{2}{ }_{21}} \hat{\mathrm{r}}_{21}$
where $\mathrm{k}=\frac{1}{4 \pi \varepsilon_{0}}$
The experimental value of the constant $\varepsilon_{0}$ is
$8.854 \times 10^{-12} \mathrm{C}^{2} \mathrm{~N}^{-1} \mathrm{~m}^{-2}$
Therefore, the approximate value of k is

$$
9 \times 10^{9} \mathrm{Nm}^{2} \mathrm{C}^{-2}
$$



Fig. Depiction of Coulomb's law

## Facts about Coulomb's law:

- Coulomb's law is not valid for charges in motion; it should only be used for point charges in vacuum at rest.
- The electrostatic force obeys Newton's third law of motion and acts along the line joining the two charges.
- Presence of other charges in the neighborhood does not affect Coulomb's force.
- The ratio of electric force and gravitational force between a proton and an electron is represented by $\frac{\mathrm{ke}^{2}}{\mathrm{Gm}_{\mathrm{e}} \mathrm{m}_{\mathrm{p}}} \cong 2.4 \times 10^{39}$


## Superposition Principle

The presence of an (or more) additional charge does not affect the forces with which two charges attract or repel each other. Superposition principle states that the net force on any charge due to $n$ number of charges at rest is the vector sum of all the forces on that charges, taken one at a time.
i.e. $\overrightarrow{\mathrm{F}}_{0}=\overrightarrow{\mathrm{F}}_{01}+\overrightarrow{\mathrm{F}}_{02}+\overrightarrow{\mathrm{F}}_{03}+. . \overrightarrow{\mathrm{F}}_{0 \mathrm{n}}$

- The force on a small positive test charge q placed at the point divided by the magnitude of the charge is the electric field $E$ at a point due to charge configuration.


## Electric Field

- The space around a charge up to which its force can be experienced is called electric field.
- Electric field due to a point charge $q$ has a magnitude $\mathrm{E}(\mathrm{r})=\frac{\mathrm{q}}{4 \pi \varepsilon_{0} \mathrm{r}^{2}} \hat{\mathrm{r}}$
$>$ It is radially outwards if $q$ is positive.
$>$ It is radially inwards if $q$ is negative.
- Electric field satisfies the superposition principle.
$>$ The unit of electric field is N/C.
$>$ Electric field inside the cavity of a charged conductor is zero.


## Electric Field lines

- The tangent at each point on the curve of electric field line, gives the direction of electric field at that point.
- The relative strength of electric field at different points is indicated by the relative closeness of field lines.
$>$ In regions of strong electric field, they crowd near each other.
$>$ In regions of weak electric field, they are far apart.
> In regions of constant electric field, the field lines formed are uniformly spaced parallel straight lines.
- Field lines are continuous curves. There will be no breaks.


Fig. Electric field lines

- Field lines are not intersecting. They cannot cross each other.
- Electrostatic field lines begin at positive charges and terminate at negative charges.
- No closed loop can be formed by them.


## Electric Dipole

- A pair of equal and opposite charges $q$ and $-q$ separated by small distance $2 a$ is known as electric dipole. The magnitude of its dipole moment vector is $2 q a$ and is in the direction of the dipole axis from $-q$ to $q$.


Fig. Electric dipole

- Field of an electric dipole in its equatorial plane at a distance $r$ from the center:

$$
\begin{aligned}
& \mathrm{E}=\frac{-\mathrm{p}}{4 \pi \varepsilon_{\mathrm{o}}} \frac{1}{\left(\mathrm{a}^{2}+\mathrm{r}^{2}\right)^{3 / 2}} \\
& \cong \frac{-\mathrm{p}}{4 \pi \varepsilon_{\mathrm{o}} \mathrm{r}^{3}} \quad \text { for } \mathrm{r} \gg \mathrm{a}
\end{aligned}
$$

- Dipole electric field on the axis at a distance r from the center:

$$
\begin{aligned}
& \mathrm{E}=\frac{2 \mathrm{pr}}{4 \pi \varepsilon_{0}\left(\mathrm{r}^{2}-\mathrm{a}^{2}\right)^{2}} \\
& \cong \frac{2 \mathrm{p}}{4 \pi \varepsilon_{0} \mathrm{r}^{3}} \quad \text { for } \mathrm{r} \gg \mathrm{a}
\end{aligned}
$$

The $1 / r^{3}$ dependence of dipole electric fields should be noted in contrast to the $1 / r^{2}$ dependence of electric field due to a point charges.

- In a uniform electric field E , a dipole experiences a torque $\tau$ given by

$$
\tau=p \times E
$$

But no net force will be experienced by it.

## PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 1

## ■ 1 Mark Questions

1. What is the geometrical shape of equipotential surface due to a single isolated charge?
[DELHI 2014]
2. Why do the electric field lines never cross each other?
[ALL INDIA 2014]
3. A point charge $+Q$ is placed at point $O$ as shown in the figure. Is the potential difference $V_{A}-V_{B}$ positive, negative or zero?

[DELHI 2016]
4. In which orientation, a dipole placed in a uniform electric field is in (i) stable, (ii) unstable equilibrium?
[DELHI 2018]
5. Draw a graph to show the variation of E with perpendicular distance $r$ from the line of charge.
6. Draw the pattern of electric field lines, when a point charge - Q is kept near an uncharged conducting plate.
[DELHI 2019]
7. How does the mobility of electrons in a conductor change, if the potential difference applied across the conductor is doubled, keeping the length and temperature of the conductor constant?
[DELHI 2019]
8. A point charge is situtated at an axial point of a small electric dipole at a large distance from it. The charge experiences a force $F$. If the distance of the charge is doubled, the force acting on the charge will become
(a) 2 F
(b) $\mathrm{F} / 2$
(c) $\mathrm{F} / 4$
(d) $\mathrm{F} / 8$
[DELHI 2020]
9. A negatively charged object $X$ is repelled by another charged object Y. However an object Z is attracted to object Y. Which of the following is the most possibility for the object Z ?
(a) Positively charged only
(b) negatively charged only
(c) neutral or positively charged
(c) neutral or negatively charged
[DELHI TERM I, 2022]
10. In an experiment three microscopic latex spheres are sprayed into a chamber and became charged with charges $+3 \mathrm{e},+5 \mathrm{e}$ and -3 e respectively. All the three spheres came in contact simultaneously for a moment and got separated. Which one of the following are possible values for the final charge on the spheres?
(a) $+5 \mathrm{e},-4 \mathrm{e},+5 \mathrm{e}$
(b) $+6 \mathrm{e},+6 \mathrm{e},-7 \mathrm{e}$
(c) $-4 \mathrm{e},+3.5 \mathrm{e},+5.5 \mathrm{e}$
(d) $+5 \mathrm{e},-8 \mathrm{e},+7 \mathrm{e}$
[DELHI TERM I, 2022]
11. An object has charge of 1 C and gains $5.0 \times 10^{18}$ electrons. The net charge on the object becomes-
(a) -0.80 C
(b) +0.80 C
(c) +1.80 C
(d) +0.20 C
[DELHI TERM I, 2022]
12. Two parallel conductors carrying current of 4.0 A and 10.0 A are placed 2.5 cm apart in vacuum. The force per unit length between them is -
(a) $6.4 \times 10^{-5} \mathrm{~N} / \mathrm{m}$
(b) $6.4 \times 10^{-2} \mathrm{~N} / \mathrm{m}$
(c) $4.6 \times 10^{-4} \mathrm{~N} / \mathrm{m}$
(d) $3.2 \times 10^{-4} \mathrm{~N} / \mathrm{m}$
[DELHI TERM I, 2022]
13. Which of the diagrams correctly represents the electric field between two charged plates if a neutral conductor is placed in between the plates?
(a)

(b)

(c)

[DELHI TERM I, 2022]
14. The magnitude of electric field due to a point charge $2 q$, at distance $r$ is $E$. Then the magnitude of electric field due to a uniformly charged thin spherical shell of radius $R$ with total charge $q$ at a distance $\frac{r}{2}(r \gg)$ will be
(a) $\frac{\mathrm{E}}{4}$
(b) 0
(c) 2 E
(d) 4 E
[DELHI TERM I, 2022]
15. Three charges $\mathrm{q},-\mathrm{q}$ and $\mathrm{q}_{0}$ are placed as shown in figure. The magnitude of the net force on the charge $q_{0}$ at point $O$ is $\left[k=\frac{1}{\left(4 \pi \epsilon_{0}\right)}\right]$

(a) 0
(b) $\frac{2 \mathrm{kqq}_{0}}{\mathrm{a}^{2}}$
(c) $\frac{\sqrt{2} \mathrm{kqq}_{0}}{\mathrm{a}^{2}}$
(d) $\frac{1}{\sqrt{2}} \frac{\mathrm{kqq}_{0}}{\mathrm{a}^{2}}$
[DELHI TERM I, 2022]
16. $A+3.0 \mathrm{nC}$ charge Q is initially at rest at a distance of $r_{1}=10 \mathrm{~cm}$ from $a+5.0 \mathrm{nC}$ charge q fixed at the origin. The charge Q is moved away from $q$ to a new position at $\mathrm{r}_{2}=15 \mathrm{~cm}$. In this process work done by the field is
(a) $1.29 \times 10^{-5} \mathrm{~J}$
(b) $3.6 \times 10^{5} \mathrm{~J}$
(c) $-4.5 \times 10^{-7} \mathrm{~J}$
(d) $4.5 \times 10^{-7} \mathrm{~J}$
[DELHI TERM I, 2022]
17. Given below are the two statements labelled as Assertion (A) and Reason (R). Select the most appropriate answer from the options given below as: Assertion (A): A negative charge in an electric field moves along the direction of the electric field.
Reason (R): On a negative charge a force acts in the direction of the electric field.
(a) Both (A) \& (R) are true and (R) is correct explanation of (A).
(b) Both (A) \& (R) are true, and (R) is not correct explanation of (A)
(c) (A) is true, but (R) is flase.
(d) (A) is false and (R) is also false.
[DELHI TERM I, 2022]
18. An electric dipole of length 2 cm is placed at an angle of $30^{\circ}$ with an electric field $2 \times 10^{5} \mathrm{~N} / \mathrm{C}$. If the dipole experiences a torque of $8 \times 10^{-3} \mathrm{Nm}$, the magnitude of either charge of the dipole is
(a) $4 \mu \mathrm{C}$
(b) $7 \mu \mathrm{C}$
(c) 8 mC
(d) 2 mC
[DELHI, 2023]

## ■ 2 Marks Question

19. An electric dipole of length 4 cm , when placed with its axis making an angle of $60^{\circ}$ with a uniform electric field, experiences a torque of $4 \sqrt{ } 3 \mathrm{Nm}$. Calculate the potential energy of the dipole, if it has charge $\pm 8 \mathrm{nC}$. [DELHI 2014]

## ■ 3 Marks Questions

20. (a) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.
(b) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop abcda.

(a) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d.
(b) Two charged spherical conductors of radii $\mathrm{R}_{1}$ and $R_{2}$ when connected by a conducting wire acquire charge $q_{1}$ and $q_{2}$ respectively. Find the ratio of their surface charge densities in terms of their radii.
[DELHI 2014]
21. A charge is distributed uniformly over a ring of radius ' $a$ '. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.
[DELHI 2016]
22. (a) Draw the equipotential surfaces corresponding to a uniform electric field in the z-direction.
(b) Derive an expression for the electric potential at any point along the axial line of an electric dipole.
[DELHI 2019]

## ■ 5 Marks Question

23. (a) Derive an expression for the electric field E due to a dipole of length " $2 a$ a at a point distant $r$ from the centre of the dipole on the axial line.
(b) Draw a graph of E versus r for $\mathrm{r} \gg \mathrm{a}$.
(c) If this dipole were kept in a uniform external electric field $\mathrm{E}_{0}$, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.
[ALL INDIA 2017]
24. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.
(b) Two identical point charges, q each are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of $Q$. [DELHI 2019]

## Solutions

1. The equipotential surfaces of an isolated charge are concentric spherical shells(co-centric shells) and potential will be inversely proportional to distance.

[1/2]
Fig. Equipotential surfaces of an isolated charge
2. If two electric fields cross each other then there would be two different values of electric field with individual directions at that location which is impossible, hence electric field lines never cross each other.
3. Potential at a distance $r$ from a given point charge Q is given by,

[1/2]

$$
\begin{aligned}
& V=\frac{1}{4 \pi \varepsilon_{o}} \frac{Q}{r} \\
& V_{A}=\frac{Q}{4 \pi \varepsilon_{o} r_{A}} \\
& V_{B}=\frac{Q}{4 \pi \varepsilon_{o} r_{B}}
\end{aligned}
$$

Since, $r_{A}<r_{B} \Rightarrow V_{A}>V_{B}$
Hence, $\mathrm{V}_{\mathrm{A}}-\mathrm{V}_{\mathrm{B}}$ is positive.
4. A dipole placed in a uniform electric field is in:
(i) Stable Equilibrium: When the electric field is directed along the direction of the dipole i.e. when $\vec{E}$ is parallel to $\vec{p}$.
(ii) Unstable Equilibrium: When the electric field is directed at an angle of $180^{\circ}$ with the direction of the dipole i.e. when $\vec{E}$ is antiparallel to $\vec{p}$.
5.


Fig: graph to show the variation of E with perpendicular distance $r$ from the line of charge.
6. Equal charge of opposite nature induces in the surface of conductor nearer to the source charge


Electric lines of forces should fall / normally $90^{\circ}$ away on / from the conducting plate.
7. If the potential difference applied across the conductor is doubled, keeping the length and temperature of the conductor, mobility of electron remains unchanged because mobility ( $\mu$ ) is independent of applied potential difference. [1]
8. (d) As electric field on axial line varies as,

$$
\begin{equation*}
\mathrm{E} \propto \frac{1}{\mathrm{r}^{3}} \text { OR } \frac{\mathrm{F}}{\mathrm{Q}} \propto \frac{1}{\mathrm{r}^{3}} \quad \text { OR } \quad \mathrm{F} \propto \frac{1}{\mathrm{r}^{3}} \tag{1}
\end{equation*}
$$

So, when distance is doubled, force reduce to $\mathrm{F} / 8$.
9. (c) (neutral or positively charged)

Since we know that like charges repel and unlike charges attract each other. If we suppose $Y$ is having negative charge and X is repelled to it, then X is also having negative charge. Also Y having negative charge, can get attracted towards positive charge, since Z is attracted to Y, so we can suppose $Z$ having positive charge. And among the options, option C satisfies being Neutral/Positive Charge.
[1]
10. (b) $+6 \mathrm{e},+6 \mathrm{e},-7 \mathrm{e}$

Given, the charges $+3 \mathrm{e},+5 \mathrm{e}$ and -3 e before being sprayed together in the container. After keeping them in the container and making them in contact, the charges get distributed equally, hence the total charge after cancelling the opposite charges of +3 e becomes +5 e. So net charge over each electron becomes +5 e . Now, calculating the sum of charges for each option shows -

| Charges |  |  | TOTAL SUM |
| :--- | :--- | :--- | :--- |
| +5 e | -4 e | +5 e | +6 e |
| +6 e | +6 e | -7 e | $+\mathbf{5 e}$ |
| -4 e | +3.5 e | +5.5 e | +13 e |
| +5 e | -8 e | +7 e | +4 e |

Hence, correct option is $(\mathrm{b})+6 \mathrm{e},+6 \mathrm{e},-7 \mathrm{e}$ [1]
11. (d) +0.20 C

Given, Number of electrons, $\mathrm{n}=5.0 \times 10^{18}$
Charge supplied, $Q_{\text {net }}=$ charge of 1 electron $x$ number of electrons
We know that, charge of 1 electron

$$
\begin{aligned}
& =-1.6 \times 10^{-19} \mathrm{C} \\
\mathrm{Q}_{\text {net }} & =\left(-1.6 \times 10^{-19} \mathrm{C}\right) \times\left(5.0 \times 10^{18}\right) \\
& =-8 \times 10^{-1}=-0.8 \mathrm{C}
\end{aligned}
$$

$$
\begin{align*}
\mathrm{Q}_{\text {finally }} & =\mathrm{Q}_{\text {net }}+\text { Initial charge } \\
& =-0.8+1=+0.20 \mathrm{C} \tag{1}
\end{align*}
$$

12. (d) $3.2 \times 10^{-4} \mathrm{~N} / \mathrm{m}$

We know $\mathrm{F}=\frac{\mu_{0} i_{1} i_{2}}{2 \pi r}$
$=\frac{4 \pi \times 10^{-7} \times 4 \times 10}{2 \pi \times 2.5 \times 10^{-2}}=3.2 \times 10^{-4} \mathrm{~N} / \mathrm{m}$
13. (d) Upper side of the neutral conductor will be negatively charged. Lower side of the neutral conductor will be positively charged. Then the field lines will be from negative to positive, as represented by figure.
14. (c) We know, the Electric Field due to a point charge $=\mathrm{E}=\frac{\mathrm{k} \times 2 \mathrm{q}}{\mathrm{r}^{2}}$
Electric field due to a spherical shell

$$
\begin{equation*}
=\mathrm{E}^{\prime}=\frac{\mathrm{k} \times 2 \mathrm{q}}{\left(\frac{\mathrm{r}}{2}\right)^{2}}=2 \mathrm{~F} \tag{1}
\end{equation*}
$$

15. (c) $\frac{\sqrt{2} \mathrm{kqq}_{0}}{\mathrm{a}^{2}}$

Positions of $q_{0},-q$ and $q$ are shown.
Both $q$ and $-q$ is equidistant from $q_{0}$.
So, the magnitude of both the forces on $q_{0}$ will be equal. The angle between the forces will be $900^{\circ}$ as in figure.
Hence the resultant force
$=\sqrt{\mathrm{F}^{2}+\mathrm{F}^{2}+2 \mathrm{~F} \times \mathrm{F} \times \cos 90^{\circ}}$
$=\sqrt{2} \mathrm{~F}=\frac{\sqrt{2 \mathrm{kqq}_{\mathrm{o}}}}{\mathrm{a}^{2}}=\sqrt{2} \times\left(\frac{1}{4 \pi \varepsilon_{0}}\right) \times \frac{q{q_{0}}^{\mathrm{a}^{2}}}{\mathrm{a}^{2}}$
16. (d) $4.5 \times 10^{-7} \mathrm{~J}$

We know that, Work Done

$$
\begin{align*}
\int_{\mathrm{r}_{1}}^{\mathrm{r}_{2}} \mathrm{~F} \mathrm{dr}=\int_{\mathrm{r}_{1}}^{\mathrm{r}_{2}} \frac{\mathrm{kqQ}}{\mathrm{r}^{2}} & \\
=\mathrm{kqQ}\left[\frac{1}{\mathrm{r}^{1}}-\frac{1}{\mathrm{r}^{2}}\right]= & \left(8.99 \times 10^{9}\right) \times\left(5 \times 10^{-9}\right) \\
& \times\left(3 \times 10^{-9}\right) \times\left[\frac{1}{0.1}-\frac{1}{0.15}\right] \tag{1}
\end{align*}
$$

$=4.5 \times 10^{4} \mathrm{C}$
17. (d) Assertion is wrong since electron moves in opposite direction of the electric field.
Reason is also false since on negative charge force acts in the opposite direction of the electric field.
18. (a) $2 \mathrm{l}=2 \mathrm{~cm}$

$\vec{\tau}=\overrightarrow{\mathrm{P}} \times \overrightarrow{\mathrm{E}}=\mathrm{PEsin} \theta$
$\Rightarrow 8 \times 10^{-3}=\mathrm{q} \times(2 \mathrm{l}) \times 2 \times 10^{5} \times \sin 30^{\circ}$
$\Rightarrow 8 \times 10^{-3}=\mathrm{q} \times 2 \times 10^{-2} \times 2 \times 10^{5} \times \frac{1}{2}$
$\Rightarrow \mathrm{q}=\frac{8 \times 10^{-3} \times 2}{4 \times 10^{3}}=4 \mu \mathrm{C}$
19. As $\tau=\mathrm{pE} \sin \theta$
$\therefore 4 \sqrt{3}=p E \sin \theta$
$\Rightarrow p E \times \frac{\sqrt{3}}{2}=4 \sqrt{3}$
$\Rightarrow \mathrm{pE}=8$
Potential energy of dipole,
$\mathrm{U}=-\mathrm{pE} \cos \theta$
$\mathrm{U}=-\mathrm{pE} \cos 60^{\circ}$
$\mathrm{U}=-4 \mathrm{~J}$
20. (a) Let us consider a parallel-plate capacitor of plate area A. If separation between plates is d metre (meter), capacitance C in given by

$$
\begin{equation*}
C=\frac{\varepsilon_{o} A}{d} F \tag{1}
\end{equation*}
$$

We know that the magnitude of the electric field between the charged plates of the capacitor in
$E=\frac{\sigma}{\varepsilon_{0}}$
Where, $\sigma$ is the surface density of either plate. Therefore, the plate charge in is $\mathrm{Q}=\sigma \mathrm{A}=\varepsilon_{0}$ EANow, the energy stored in the capacitor in $U=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} \frac{\left(\varepsilon_{0} E A\right)^{2}}{\varepsilon_{0} A / d}$
$U=\frac{1}{2} \varepsilon_{o} E^{2}(A d) J$
The volume between the plates is $A d$ metre ${ }^{3}$. Therefore, the energy per unit volume is given by,
$\boldsymbol{U}=\frac{\boldsymbol{U}}{A d}=\frac{1}{2} \varepsilon_{o} E^{2} J / m^{3}$


Fig.: Parallel plate capacitor
(b) Work done, $\boldsymbol{W}=\boldsymbol{F} \cdot \boldsymbol{d}$

Here, $F$ is the exerted on the charge (q) due to electric field (E) and is given by, $F=q E$
Net displacement, $d=0$
Hence, W = 0
OR
(a) Derivation for the capacitance of parallel plate capacitor:


Surface charge density $-\sigma$
[1/2]
Fig. Capacitance of a parallel plate capacitor
A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance d. The two plates have charges $q$ and -1 and distance between them is d .
Plate 1 has charge density $\sigma=\frac{q}{\mathrm{~A}}$
Plate 2 has charge density $\sigma=-\frac{\mathrm{q}}{\mathrm{A}}$
In the inner region between the plates 1 and 2 , the electric fields due to the two charged plates add up
$E=\frac{q}{2 \varepsilon_{0}}+\frac{q}{2 \varepsilon_{0}}=\frac{q}{\varepsilon_{0}}=\frac{q}{A \varepsilon_{0}}$
[1/2]
For this electric field, potential difference between the plates in given by,
$\mathrm{V}=\mathrm{Ed}=\frac{1}{\varepsilon_{0}} \frac{\mathrm{qd}}{\mathrm{A}}$
The capacitance $C$ of the parallel plate capacitor is then, $C=\frac{Q}{V}=\frac{\varepsilon_{0} A}{d}$
(b) The surface charge density for a spherical conductor of radius $R_{1}$ is given by:
$\sigma=\frac{\mathrm{q}_{1}}{4 \pi \mathrm{R}_{1}^{2}}$
Similarly, for spherical conductor $R_{2}$, the surface charge density is given by:
$\sigma_{2}=\frac{q_{2}}{4 \pi R_{2}^{2}}$
$\frac{\sigma_{1}}{\sigma_{2}}=\frac{q_{1} R_{2}^{2}}{q_{2} R_{1}^{2}}$

As the spheres are connected so the charges will flow between the spherical conductors till their potential become equal.

$$
\begin{align*}
& \frac{k q_{1}}{R_{1}}=\frac{k q_{2}}{R_{2}}  \tag{1/2}\\
& \frac{q_{1}}{R_{1}}=\frac{q_{2}}{R_{2}} \tag{2}
\end{align*}
$$

Using (2) in (1) We have,

$$
\begin{align*}
& \frac{\sigma_{1}}{\sigma_{2}}=\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}} \cdot \frac{\mathrm{R}_{2}^{2}}{\mathrm{R}_{1}^{2}} \Rightarrow \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \\
& \frac{\sigma_{1}}{\sigma_{2}}=\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \tag{1/2}
\end{align*}
$$

21. Suppose we have a ring of radius a that carries a uniformly distributed positive charge q.


Fig. Uniform distribution of a charge over a ring
[1/2]
As the total charge $q$ is uniformly distributed, the charge dq on the element dl is
$d q=\frac{q}{2 \pi \alpha} \cdot d l$
$\therefore$ The magnitude of the electric field produced by the element dl at the axial point P is

$$
d E=k \cdot \frac{d q}{r^{2}}=\frac{k q}{2 \pi a} \cdot \frac{d l}{r^{2}}
$$

(i) The axial components $\mathrm{dE} \cos \theta$ and
(ii) The perpendicular component dEsin$\theta$.
[1/2]
Since the perpendicular component of any two diametrically opposite elements are equal and opposite, they cancel out in pairs. Only the axial components will add up to produce the resultant field.

E at point P is given by, $E=\int_{0}^{2 \pi a} d E \cos \theta$
( $\because$ Only the axial components contribute towards E )
$E=\int_{0}^{2 \pi a} \frac{k q}{2 \pi a} \cdot \frac{d l}{r^{2}} \cdot \frac{x}{r}$
$E=\frac{k q x}{2 \pi a} \cdot \frac{1}{r^{3}} \int_{0}^{2 \pi a} d l$
$\left(\because \cos \theta=\frac{\mathrm{x}}{\mathrm{r}}\right)$
$E=\frac{k q x}{2 \pi a} \cdot \frac{1}{r^{3}}(l)_{o}^{2 \pi a}$
$E=\frac{k q x}{2 \pi a} \cdot \frac{1}{\left(x^{2}+a^{2}\right)^{3 / 2}} \cdot 2 \pi a$
$\because r^{2}=x^{2}+a^{2}$
$E=\frac{k q x}{\left(x^{2}+a^{2}\right)^{3 / 2}}$
[1/2]

If $x \gg a$, then $x^{2}+a^{2} \approx x^{2}$
$\mathrm{E}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \frac{\mathrm{qx}}{\left(\mathrm{x}^{2}\right)^{3 / 2}}$
$\mathrm{E}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \frac{\mathrm{q}}{\mathrm{x}^{2}}$
This expression is similar to electric field due to point charge.
22. (a) Equipotential surfaces due to an electric dipole :

(b) Let distance of point p where field has to be calculate be y from axial line,


$\therefore \overrightarrow{\mathrm{E}}=\overrightarrow{\mathrm{E}}_{-\mathrm{q}}+\overrightarrow{\mathrm{E}}_{+\mathrm{q}}$
Due to symmetry electric field in y direction will cancel out.
$\overrightarrow{\mathrm{E}}=2\left|\overrightarrow{\mathrm{E}}_{-\mathrm{q}}\right| \cos \theta(-\hat{\mathrm{i}})=\frac{2 \mathrm{qa}(-\hat{\mathrm{i}})}{4 \pi \varepsilon_{0}\left(\mathrm{a}^{2}+\mathrm{y}^{2}\right)^{3 / 2}}$
for $\mathrm{y} \gg \mathrm{a}$
$\therefore \overrightarrow{\mathrm{E}}=\frac{2 \mathrm{qa}}{4 \pi \varepsilon_{0} \mathrm{y}^{3}}(-\hat{\mathrm{i}})=-\frac{\overrightarrow{\mathrm{p}}}{4 \pi \varepsilon_{0} \mathrm{y}^{3}}$
23.


Electric field intensity due to as electric dipole
(a) Dipole at a point on the axial wire: we have to a calculate the field intensity (E) at a point $P$ on the axial line of the dipole and dt a distance $\mathrm{OP}=\pi$ from the centre O of the dipole. Resultant electric field intensity at the point $P, E_{P}=E_{A}+E_{B}$ The vectors $E_{A}$ and $\mathrm{E}_{\mathrm{B}}$ are collinear at opposite.
$\therefore \mathrm{E}_{\mathrm{P}}=\mathrm{E}_{\mathrm{A}}-\mathrm{E}_{\mathrm{B}}$
Here, $\mathrm{E}_{\mathrm{A}}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{(\mathrm{x}-\mathrm{l})^{2}}$ and $\mathrm{E}_{\mathrm{B}}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{(\mathrm{x}+1)^{2}}$

Thus,
$E_{p}=\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{q}{(x-l)^{2}}-\frac{q}{(x+l)^{2}}\right]=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{4 q l x}{\left(x^{2}-l^{2}\right)^{2}}$
Hence, $E_{p}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{4 p x}{\left(x^{2}-l^{2}\right)^{2}}[\therefore p=q \times 2 l] \quad[1 / 2]$
In vector form, $E_{p}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{4 p x}{\left(x^{2}-l^{2}\right)^{2}}$
If the dipole is short,i.e., $21 \ll \mathrm{x}$, then
$E_{p}=\frac{2}{4 \pi \varepsilon_{0}} \cdot \frac{|P|}{x^{3}}$
The direction of $\mathrm{E}_{\mathrm{P}}$ is long BP produced clearly,
$E_{P} \propto \frac{1}{x^{3}}$
(b) Graph of E versus r for $\mathrm{r} \gg \mathrm{a}$


Fig.: E versus r
(c) Torque on an electric dipole in uniform electric field :-

[1/2]
Consider an electric dipole considering of two changes $-q$ and $+q$ placed is a uniform external electric field of intensity E. The dipole moment $P$ makes an angle $\theta$ with the direction of the electric field. The net force is zero. Since, the two forces are equal in magnitude and opposite in direction and act at different points therefore they constitute a couple. A net torque $\tau$ acts
on the dipole about an axis passing through the mid-point of the couple. Now $\tau=$ force $\times$ perpendicular distance BC between the parallel force $\mathrm{qE}(2 l \sin \theta)$
$\tau=(\theta \times 2 \mathrm{l}) \mathrm{E} \sin \theta$ or $\tau=\mathrm{pE} \sin \theta$
[1/2]
In vector notation, $\tau=\mathrm{p} \times \mathrm{E}$
SI unit of torque is newton-meter ( $\mathrm{N}-\mathrm{m}$ ) and its dimensional formula is $\left[\mathrm{ML}^{2} \mathrm{~T}^{2}\right]$
Case-I: If $\theta=0^{\circ}$ then $\tau=0$,
The dipole is in stable equilibrium Case-II: If $\theta=90^{\circ}$, then $\tau=\mathrm{PE}$ (maximum value) The torque acting on dipole will be maximum.
Case-III: If $\theta=180^{\circ}$ then $\tau=0$
The dipole is in unstable equilibrium
24. (a) Let P be an equatorial point for a dipole consisting of charges $-q$ and $+q$ with a separation 2 a , then $\overrightarrow{\mathrm{E}}_{+}=$electric field due to the charge $+q$


$$
\begin{equation*}
\overrightarrow{\mathrm{E}}_{+}=\frac{1}{4 \pi \varepsilon_{0}} \times \frac{\mathrm{q}}{\mathrm{a}^{2}+\mathrm{r}^{2}} \tag{1}
\end{equation*}
$$

Again,
$\overrightarrow{\mathrm{E}}=$ electric field due to the charge -q

$$
\begin{equation*}
=\frac{1}{4 \pi \varepsilon_{0}} \times \frac{\mathrm{q}}{\mathrm{a}^{2}+\mathrm{r}^{2}} \tag{ii}
\end{equation*}
$$

By superposition principle,

$$
\begin{aligned}
& \overrightarrow{\mathrm{E}}_{\text {net }}=\overrightarrow{\mathrm{E}}_{+}+\overrightarrow{\mathrm{E}}_{-} \\
& \therefore\left|\overrightarrow{\mathrm{E}}_{\text {net }}\right|=2 \mathrm{E}_{+} \cos \theta=\frac{2}{4 \pi \varepsilon_{0}} \times \frac{\mathrm{q}}{\left(\mathrm{a}^{2}+\mathrm{r}^{2}\right)} \times \frac{\mathrm{a}}{\sqrt{\mathrm{a}^{2}+\mathrm{r}^{2}}}
\end{aligned}
$$

$$
\begin{equation*}
=\frac{1}{4 \pi \varepsilon_{0}} \times \frac{2 \mathrm{aq}}{\left(\mathrm{a}^{2}+\mathrm{r}^{2}\right)^{3 / 2}} \tag{1}
\end{equation*}
$$

In vector form,

$$
\overrightarrow{\mathrm{E}}_{\mathrm{net}}=\frac{1}{4 \pi \varepsilon_{0}} \times \frac{-\overrightarrow{\mathrm{p}}}{\left(\mathrm{a}^{2}+\mathrm{r}^{2}\right)^{3 / 2}}
$$

For short dipole, $r \gg$ a, then
$\overrightarrow{\mathrm{E}}_{\text {net }}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{-\overrightarrow{\mathrm{p}}}{\mathrm{r}^{3}}\right)$
[1]
(b) To overcome the electrostatic repulsion between the point charges $q$ each, $Q$ must be at the centre of the line segment joining the two. The sign of $Q$ must be opposite to that of $q$.

$\Rightarrow \frac{\mathrm{q}}{4}=-\mathrm{Q}$
$\therefore \mathrm{Q}=-\frac{\mathrm{q}}{4}$
Hence, $Q$ must be at a distance of 1 m from each charge q.

## MULTIPLE CHOICE QUESTIONS

1. When the distance between the charged particles is halved, the force between them becomes.
(a) One-fourth
(b) Half
(c) Double
(d) Four times
2. A charge $q_{1}$ exerts some force on a second charge $q_{2}$. If third charge $q_{3}$ is brought near, the force of $q_{1}$ exerted on $q_{2}$.
(a) Decreases
(b) Increases
(c) Remains unchanged
(d) Increases if $q_{3}$ is of the same signs as $q_{1}$ and decreases if $q_{3}$ is of opposite sign
3. The minimum charge on an object is
(a) 1 coulomb
(b) 1 stat coulomb
(c) $1.6 \times 10^{-19}$ coulomb
(d) $3.2 \times 10^{-19}$ coulomb
4. Three charges $4 q, Q$ and $q$ are in a straight line in the position of $0, \mathrm{l} / 2$ and 1 respectively. The resultant force on $q$ will be zero, if $Q=$
(a) -q
(b) $-2 q$
(c) $-\frac{\mathrm{q}}{2}$
(d) $4 q$
5. The number of electrons in 1.6 C charge will be
(a) $10^{19}$
(b) $10^{20}$
(c) $1.1 \times 10^{19}$
(d) $1.1 \times 10^{2}$
6. The electric charge in uniform motion produces
(a) An electric field only
(b) A magnetic field only
(c) Both electric and magnetic field
(d) Neither electric nor magnetic field
7. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and $B$ are $E_{A}$ and $E_{B}$ respectively and if the displacement between $A$ and $B$ is $r$, then

(a) $\mathrm{E}_{\mathrm{A}}>\mathrm{E}_{\mathrm{B}}$
(b) $\mathrm{E}_{\mathrm{A}}<\mathrm{E}_{\mathrm{B}}$
(c) $\mathrm{E}_{\mathrm{A}}=\frac{\mathrm{E}_{\mathrm{B}}}{\mathrm{r}}$
(d) $\mathrm{E}_{\mathrm{A}}=\frac{\mathrm{E}_{\mathrm{B}}}{\mathrm{r}^{2}}$
8. The electric field near a conducting surface having a uniform surface charge density $\sigma$ is given by
(a) $\frac{\sigma}{\varepsilon_{0}}$ and is parallel to the surface
(b) $\frac{2 \sigma}{\varepsilon_{0}}$ and is parallel to the surface
(c) $\frac{\sigma}{\varepsilon_{0}}$ and is normal to the surface
(d) $\frac{2 \sigma}{\varepsilon_{0}}$ and is normal to the surface
9. Deutron and $\alpha$-particle are put $1 \AA$ apart in air. Magnitude of intensity of electric field due to deutron at $\alpha$-particle is
(a) zero
(b) $2.88 \times 10^{11} \mathrm{~N} / \mathrm{C}$
(c) $1.44 \times 10^{11} \mathrm{~N} / \mathrm{C}$
(d) $5.76 \times 10^{11} \mathrm{~N} / \mathrm{C}$
10. An electric dipole when placed in a uniform electric field E will have minimum potential energy, if the positive direction of dipole moment makes the following angle with E
(a) $\pi$
(b) $\frac{\pi}{2}$
(d) zero
(d) $\frac{3 \pi}{2}$
11. The electric potential at a point on the axis of an electric dipole depends on the distance $r$ of the point from the dipole as
(a) $\propto \frac{1}{\mathrm{r}}$
(b) $\propto \frac{1}{\mathrm{r}^{2}}$
(c) $\propto \mathrm{r}$
(d) $\propto \frac{1}{\mathrm{r}^{3}}$
12. An electric dipole is kept in non-uniform electric field. It experiences
(a) A force and a torque
(b) A force but not a torque
(c) A torque but not a force
(d) Neither a force nor a torque
13. The distance between the two charges $+q$ and $-q$ of a dipole is $r$. On the axial line at a distance $d$ from the centre of dipole, the intensity is proportional to
(a) $\frac{\mathrm{q}}{\mathrm{d}^{2}}$
(b) $\frac{\mathrm{qr}}{\mathrm{d}^{2}}$
(c) $\frac{\mathrm{q}}{\mathrm{d}^{3}}$
(d) $\frac{\mathrm{qr}}{\mathrm{d}^{3}}$
14. The electric field due to an electric dipole at a distance $r$ from its centre in axial position is E . If the dipole is rotated through an angle of $90^{\circ}$ about its perpendicular axis, the electric field at the same point will be
(a) E
(b) $\frac{\mathrm{E}}{4}$
(c) $\frac{\mathrm{E}}{2}$
(d) 2 E
15. An electric dipole of moment $\vec{\rho}$ placed in a uniform electric field $\overrightarrow{\mathrm{E}}$ has minimum potential energy when the angle between $\vec{\rho}$ and $\overrightarrow{\mathrm{E}}$ is
(a) Zero
(b) $\frac{\pi}{2}$
(c) $\pi$
(d) $\frac{3 \pi}{2}$

Answer Keys

1. (d)
2. (c)
3. (c)
4. (a)
5. (a)
6. (c)
7. (a)
8. (c)
9. (c)
10. (c)
11. (d)
12. (a)
13. (d)
14. (c)
15. (a)

## Solutions

1. $\because \mathrm{f} \propto \frac{1}{\mathrm{r}^{2}}$
$\therefore$ when r is halved the force becomes four times.
2. The force will still remain unchanged.
$\therefore \mathrm{F}=\frac{\mathrm{q}_{1} \mathrm{q}_{2}}{4 \pi \varepsilon_{0} \mathrm{r}^{2}}$
3. All other charges are its integral multiple.
$\therefore$ Minimum charge on an object $=1.6 \times 10^{-19}$ coulomb
4. The force between $4 q$ and $q$.
$\mathrm{F}_{1}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{4 \mathrm{q} \times \mathrm{q}}{\mathrm{l}^{2}}$

The force between $Q$ and $q$
$\mathrm{F}_{2}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{Q} \times \mathrm{q}}{\left(\frac{1}{2}\right)^{2}}$
$\therefore \mathrm{F}_{1}+\mathrm{F}_{2}=0$ or $\frac{4 \mathrm{q}^{2}}{\mathrm{l}^{2}}=-\frac{4 \mathrm{Qq}}{\mathrm{l}^{2}} \Rightarrow \mathrm{Q}=-\mathrm{q}$
5. $\mathrm{n}=\frac{\mathrm{q}}{\mathrm{e}}=\frac{1.6}{1.6 \times 10^{-19}}=10^{19}$
6. A movable charge produces electric field and magnetic field both.
7. In non-uniform electric field. Intensity is more, where the lines are more denser.
8. Electric field near the conductor surface is given by $\frac{\sigma}{\varepsilon_{0}}$ and it is perpendicular to surface.
9. Due to deutron, intensity of electric field at $1 \AA$ distance
$\mathrm{E}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{e}}{\mathrm{r}^{2}}=\frac{9 \times 10^{9} \times 1.6 \times 10^{-19}}{10^{-20}}$
$=1.44 \times 10^{11} \mathrm{~N} / \mathrm{C}$
10. Potential energy $=-\mathrm{pE} \cos \theta$.
when $\theta=0$,
Potential energy $=-\mathrm{pE}$ (minimum)
11. Electric potential due to dipole in it's general position is given by $\mathrm{v}=\frac{\mathrm{k} \cdot \mathrm{p} \cos \theta}{\mathrm{r}^{2}} \Rightarrow \mathrm{v} \propto \frac{1}{\mathrm{r}^{2}}$
12. As the dipole will feel two forces which are although opposite but not equal.
$\therefore$ A net force will be there and as these forces act at different points of a body. A torque is also parent.
13. Field along the axis of the dipole

$$
\begin{align*}
& \mathrm{E}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{2 \mathrm{p}}{\mathrm{~d}^{3}}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{2(\mathrm{q} \times \mathrm{r})}{\mathrm{d}^{3}} \\
& \therefore \mathrm{E} \propto \frac{\mathrm{qr}}{\mathrm{~d}^{3}} \tag{1}
\end{align*}
$$

14. When the dipole is rotated through at an angle of $90^{\circ}$ about it's perpendicular axis then given point comes out to be on equator. So field will become $\frac{\mathrm{E}}{2}$ at the given point.
15. $\mathrm{U}=-\mathrm{PE} \cos \theta$.

It has minimum value when $\theta=0^{\circ}$.
i.e. $\mathrm{U}_{\min }=-\mathrm{PE} \cos 0^{\circ}=-\mathrm{PE}$

## Topic 2: Electric Flux

## Summary

- Electric flux is proportional to number of lines leaving a surface, outgoing lines with positive sign, incoming lines with negative sign.


Fig. Electric flux

- Through a small area element $\Delta \mathrm{S}$, the flux $\Delta \phi$ of electric field E is given by
$\Delta \phi=\mathrm{E} . \Delta \mathrm{S}$
And the vector area element $\Delta \mathrm{S}$ is
$\Delta S=\Delta S \hat{n}$
Where $\Delta \mathrm{S}$ is the magnetic of the area element and $\hat{\mathrm{n}}$ is normal to the area element, which can be considered planar for the sufficiently small $\Delta \mathrm{S}$.


## Gauss's Law and its application

- The flux of electric field through any closed surface $S$ is $1 / \varepsilon_{0}$ times the total charge enclosed by S .

$$
\phi=\mathrm{E} \int \mathrm{dA}=\frac{\mathrm{q}_{\text {enclosed }}}{\varepsilon_{0}}
$$

- The law is mainly useful in determining electric field $E$, when the source distribution has simple symmetry:
> Thin infinitely long straight wire of uniform linear charge density $\lambda$


Fig. Thin infinitely long Straight wire

$$
\mathrm{E}=\frac{\lambda}{2 \pi \varepsilon_{0} \mathrm{r}} \hat{\mathrm{n}}
$$

Where, $r$ is the radial (perpendicular) distance of the point from the wire and $\hat{\mathrm{n}}$ is the radial unit vector in
the plane normal to the wire passing through the point.

- Infinite plane sheet (thin) of uniform surface charge density $\sigma$


Fig. Infinite plane sheet (thin)

$$
E=\frac{\sigma}{2 \varepsilon_{o}} \hat{n}
$$

Where $\hat{\mathrm{n}}$ is a unit vector normal to the plane and going away from it.

- Thin spherical shell of uniform surface charge density $\sigma$

$$
\begin{aligned}
& \mathrm{E}=\frac{\mathrm{q}}{4 \pi \varepsilon_{0} \mathrm{r}^{2}} \hat{\mathrm{r}} \quad(\mathrm{r} \geq \mathrm{R}) \\
& \begin{array}{l}
\text { Surface charge } \\
\text { density } \\
\end{array}
\end{aligned}
$$

Fig.: Thin uniformly surface charged spherical shell ( $\mathrm{r}>\mathrm{R}$ )
(For $r>R$ )
$\mathrm{E}=0(\mathrm{r}<\mathrm{R})$


Fig.: Thin uniformly surface charged spherical

$$
\text { shell }(\mathrm{r}<\mathrm{R})
$$

(For $\mathrm{r}<\mathrm{R}$ )
Where $r$ is the distance of the point from the center of the shell whose radius is $R$ with the total charge q. The electric field outside the shell is the same as the total charge is concentrated at the center. A solid sphere of uniform volume charge density shows the same result. Inside the shell at all the points, the field is zero.

## PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 2

## ■ 1 Mark Questions

1. What is the electric flux through a cube of side 1 cm which encloses an electric dipole?
[DELHI 2015]
2. Figure shows three point charges $+2 q,-q$ and $+3 q$. Two charges $+2 q$ and $-q$ are enclosed within a surface ' S '. What is the electric flux due to this configuration through the surface ' S '?
[DELHI 2015]

3. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased?
[DELHI 2016]
4. If the net electric flux through a closed surface is zero, then we can infer
(a) no net charge is enclosed by the surface.
(b) uniform electric field exists within the surface.
(c) electric potential varies from point to point inside the surface.
(d) charge is present inside the surface.
[DELHI 2020]
5. A square sheet of side 'a' is lying parallel to XY plane at $\mathrm{z}=\mathrm{a}$. The electric field in the region is $\overrightarrow{\mathrm{E}}=c z^{2} \hat{\mathrm{k}}$. The electric flux through the sheet is
(a) $a^{4} c$
(b) $\frac{1}{3} \mathrm{a}^{3} \mathrm{c}$
(c) $\frac{1}{3} a^{4} c$
(d) 0
[DELHI TERM I, 2022]

## ■ 2 Marks Questions

6. Given a uniform electric field $\vec{E}=5 \times 10^{3} \hat{i} N / C$, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the $\mathrm{y}-\mathrm{z}$ plane. What would be the flux through the same square if the plane makes a $30^{\circ}$ angle with the x-axis?
[DELHI 2014]
7. Given a uniform electric field $\vec{E}=2 \times 10^{3} \hat{i} N / C$. Find the flux of this field through a square of side 20 cm , whose plane is parallel to the $y-z$ plane. What would be the flux through the same square, if the plane makes an angle of $30^{\circ}$ with the x -axis?
[DELHI 2014]
8. Given a uniform electric field $\vec{E}=4 \times 10^{3} \hat{i} N / C$, find the flux of this field through a square of 5 cm on a side whose plane is parallel to the $\mathrm{y}-\mathrm{z}$ plane. What would be the flux through the same square, if the plane makes an angle of $30^{\circ}$ with the x -axis?
[DELHI 2014]

## ■ 3 Marks Question

9. Using Gauss's law to obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius $R$ at a point outside the shell. Draw a graph showing the variation of electric field with $r$, for $r>R$ and $r<R$.
[ALL INDIA 2011]

## ■ 5 Marks Questions

10. (a) An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance 2a apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on ts axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit
(b) Given the electric field in the region $\vec{E}=2 \hat{x} l$, find the net electric flux though the cube and the charge enclosed by it.

(a) Explain, using suitable diagrams, the difference in the behavior of a (i) conductor and (ii) a dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.
(b) A thin metallic spherical shell of radius a carries a charge $Q$ on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and another charge $+2 Q$ is placed outside the shell at a distance $x$ from the centre as
shown in the figure. Find (i) the force on the charge at the centre of shell and at the point A, (ii) the electric flux through the shell.
[DELHI 2015]
11. (a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet.
(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Find the amount of work done in bringing a point charge q from infinity to a point, distance $r$, in front of the charged plane sheet
[ALL INDIA 2017]
12. (a) Define electric flux. Is it a scalar or a vector quantity? A point charge $q$ is at a distance of
$\frac{d}{2}$ directly above the centre of a square of side
' $d$ ', as shown in the figure. Use Gauss's theorem to obtain the expression for the electric flux through the square.

(b) If the point charge is now moved to a distance ' $d$ ' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

## OR

Use Gauss' law to derive the expression for the electric field $(\vec{E})$ due to a straight uniformly charged infinite line of charge density $\lambda \mathrm{C} / \mathrm{m}$.
[ALL INDIA 2018]
13. (a) Using Gauss law, derive expression for electric field due to a spherical shell of uniform charge distribution $\sigma$ and radius R at a point lying at a distance $x$ from the centre of shell, such that
(i) $0<x<R$, and
(ii) $x>R$
(b) An electric field is uniform and acts along $+x$ direction in the region of positive $x$. It is also uniform with the same magnitude but acts in $-x$ direction in the region of negative $x$. The value of the field is $E=200$ N/C for $\mathrm{x}>0$ and $\mathrm{E}=-200 \mathrm{~N} / \mathrm{C}$ for $\mathrm{x}<0$. A right circular cylinder of length 20 cm
and radius 5 cm has its centre at the origin and its axis along the x -axis so that one flat face is at $x=+10 \mathrm{~cm}$ and the other is at $\mathrm{x}=-10 \mathrm{~cm}$.
Find :
(i) The net outward flux through the cylinder.
(ii) The net charge present inside the cylinder.
[DELHI 2020]

## Solutions

1. From Gauss law the net flux passing through a surface is proportional to the charge enclosed within the surface. Since , net charge enclosed by electric dipole is zero hence flux will be zero. [1]
2. From gauss law net flux is ratio of total charge enclosed divided by $(S)=\frac{q}{\varepsilon_{0}}$ from the figure total charge enclosed is $+2 q-q=q$. Hence

$$
\begin{equation*}
(S)=\frac{q}{\varepsilon_{0}} \tag{1}
\end{equation*}
$$

3. According to Gauss's law, $\phi=\int \varepsilon \cdot d s=\frac{q_{e n}}{\varepsilon_{0}}[1 / 2]$

Where $q_{\text {en }}$ is the total charge enclosed by the surface. From above formula it is clear that electric flux does not depend on radius, hence it remains constant.
Flux depends only on the charge enclosed.
Hence, the electric flux remains constant.
4. (a) No net charge is enclosed by the surface, if the net electric flux through a closed surface is zero.
5. (a) $\mathrm{E}=\mathrm{cz}^{2} \mathrm{k}$

Where, $\mathrm{z}=\Phi,=$ ?
We know that, $\varphi=\int$ E.ds
$\mathrm{ds}=\mathrm{dxdyk}$
$\varphi=\int\left(\mathrm{ca}^{2}\right) \mathrm{kdxdyk}=\int_{0}^{\mathrm{Z}} \mathrm{ca}{ }^{2} \mathrm{dxdy}$
$=\int_{0}^{z} c^{2} d x=\int_{0}^{z} c a^{2} d y=c a^{2}[a][a]=c a^{4}$
6. When the plane is parallel to the y-z plane:

Electric flux, $\phi=$ EA
Here, $\vec{E}=5 \times 10^{3} j N / C$
$A=10 \mathrm{~cm}^{2}, \hat{i}=10^{-2} \hat{i m}^{2}=10^{-2} \mathrm{im}^{2}$
$\phi=5 \times 10^{3} \hat{\mathrm{i}} 10^{-2} \hat{\mathrm{i}} \Rightarrow \phi=50$ Weber or $\mathrm{Nm}^{2} \mathrm{C}^{-1}$
When the plane makes a $30^{\circ}$ angle with the x -axis, the area vector makes $60^{\circ}$ with the x -axis.
$\phi=\overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{A}} \Rightarrow \phi=\mathrm{EA} \cos \theta$
$\phi=5 \times 10^{3} \times 10^{-2} \cos 60^{\circ}$
$\phi=\frac{50}{2}$
$\Rightarrow \phi=25$ Weber or $\mathrm{Nm}^{2} \mathrm{C}^{-1}$
7. When the plane is parallel to the y-z plane:

$$
\begin{align*}
& \phi=\vec{E} \cdot \vec{A} \\
& \vec{E}=2 \times 10^{3} \hat{i} \\
& A=(20 \mathrm{~cm})^{2} \hat{i}=0.04 M^{2} \hat{i} \\
& A=(20 \mathrm{~cm})^{2} \mathrm{i}=\left(20 \times 10^{-2}\right)^{2}=0.04 \mathrm{~m}^{2} \mathrm{i} \\
& \therefore \phi=\left(2 \times 10^{3} \hat{i}\right) \cdot(0.04 \hat{i}) \Rightarrow \phi=82 \tag{1}
\end{align*}
$$

Weber or $80 \mathrm{Nm}^{2} \mathrm{C}^{-1}$
When the plane makes an $30^{\circ}$ angle with the x -axis, the area vector makes an $60^{\circ}$ angle with the x-axis.
$\phi=\vec{E} \cdot \vec{A} \Rightarrow \phi=E A \cos \theta$
$\phi=2 \times 10^{3} \times 0.04 \cos 60^{\circ}$
$\phi=2 \times 10^{3} \times 0.04 \cos 30^{\circ}$
$\phi=2 \times 10^{3} \times 0.04 \times \frac{1}{2}$
$\Rightarrow \phi=40$ Weber or $40 \mathrm{Nm}^{2} \mathrm{C}^{-1}$
8. When the plane is parallel to the y-z plane:

Electric flux, $\phi=\vec{E} \cdot \vec{A}$
Here, $\vec{E}=4 \times 10^{3} \hat{i} N / C$
$\vec{A}=(5 \mathrm{~cm})^{2} \hat{i}=0.25 \times 10^{-2} \hat{i m}^{2}$
$\phi=\left(4 \times 10^{3} \hat{i}\right) \cdot\left(25 \times 10^{-4} \hat{i}\right)$
$\Rightarrow \phi=10$ Weber or $\mathrm{Nm}^{2} \mathrm{C}^{-1}$
When the plane makes an angle of $30^{\circ}$ with the $x$-axis, the area vector makes an angle of $60^{\circ}$ with the x-axis.
$\phi=\vec{E} \cdot \vec{A} \Rightarrow \phi=E A \cos \theta$
$\Rightarrow \phi=\left(4 \times 10^{3}\right)\left(25 \times 10^{-4}\right) \cos 60^{\circ}$
$\Rightarrow \phi=\frac{10}{2}$
$\Rightarrow \phi=5$ Weber or $\mathrm{Nm}^{2} \mathrm{C}^{-1}$
9.


Fig.: Spherical Gaussian surface
Consider a spherical Gaussian surface of radius $r(\curvearrowright R)$, concentric with given shell. If $E$ is electric field outside the shell, then by symmetry, electric field strength has same magnitude $\mathrm{E}_{\text {o }}$ on the Gaussian surface and is directed radially outward. Also the direction of normal at each point is radially outward, so angle between $\mathrm{E}_{\mathrm{o}}$ and ds is zero at each point. Hence, electric flux through Gaussian surface
[1/2]
$=\phi_{\mathrm{s}} \mathrm{E}_{\mathrm{o}} \mathrm{ds}$
$=\phi_{\mathrm{s}} \mathrm{E}_{\mathrm{o}} \mathrm{ds} \cos 0^{\circ}=\mathrm{E}_{\mathrm{o}} 4 \pi \mathrm{r}^{2} \mathrm{ds}$
Now, Gaussian surface is outside the given charged shell, so charge enclosed by the Gaussian surface is Q. Hence, by Gauss's theorem
$\phi_{\mathrm{S}} \mathrm{E}_{\mathrm{o}} \cdot \mathrm{ds}=\frac{1}{\varepsilon_{0}} \times$ charge - enclosed
Add sign of integration in this formula
$\Rightarrow \mathrm{E}_{\mathrm{o}} \cdot 4 \pi \mathrm{r}^{2}=\frac{1}{\varepsilon_{\mathrm{o}}} \times \mathrm{Q}$
$\Rightarrow E_{o}=\frac{1}{4 \pi \varepsilon_{o}} \frac{Q}{r^{2}}$
Thus, electric field outside a charged thin spherical shell is same as if the whole charge Q is concentrated at the centre. Graphically,


E is proportional to $1 / r^{2}$ not multiple as shown in the figure.

For $\boldsymbol{r}<\boldsymbol{R}$, there is no strength of electric field inside a charged spherical shell. For $\boldsymbol{r}>\boldsymbol{R}$, electric field outside a charged thin spherical shell is same as if the whole charge $Q$ is concentrated at the centre.
10. (a) Electric field at a point on the axial line

$$
\begin{aligned}
& \left|\overrightarrow{E_{+q}}\right|=\frac{k q}{(x-a)^{2}}\left|\overline{E_{-q}}\right|=\frac{k q}{(x+a)^{2}} \\
& \qquad\left[\because \mathrm{k}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}}\right] \\
& {[\because \vec{P}=2 a q]}
\end{aligned}
$$

If $x \ggg a$,
In vector form, $\overrightarrow{\mathrm{E}}=\frac{2 \mathrm{p}}{4 \pi \varepsilon_{0} \mathrm{x}^{3}}$
(b) Since, the electric field is parallel to the faces parallel to $x y$ and $x z$ planes, the electric flux through them is zero.
Electric flux through the left face,
$\phi_{\mathrm{L}}=\left(\mathrm{E}_{\mathrm{L}}\right)\left(\mathrm{a}^{2}\right) \cos 180^{\circ}$
$\phi_{\mathrm{L}}=(0)\left(\mathrm{a}^{2}\right) \cos 180^{\circ}=0$

Electric flux through the right face,
$\phi_{\mathrm{R}}=\left(\mathrm{E}_{\mathrm{R}}\right)\left(\mathrm{a}^{2}\right) \cos 0^{\circ}$
$\phi_{R}=(2 a)\left(a^{2}\right) \times 2 a^{3}$
Total flux $(\phi)=2 \mathrm{a}^{3}=\frac{q_{\text {enclosed }}}{\varepsilon_{0}}$
$\therefore \mathrm{q}_{\text {enclosed }}=2 \mathrm{a}^{3} \varepsilon_{0}$
OR

(a) (i) Conductor $\mathrm{E}_{\mathrm{o}} \rightarrow$ External field
$\mathrm{E}_{\mathrm{m}} \rightarrow$ Internal field created by the redistribution of electrons inside the metal
When a conductor like a metal is subjected to external electric field, the electrons experience a force in the opposite direction collecting on the left side.
A positive charge is therefore induced on the right hand side. This creates an opposite electric field $\left(\mathrm{E}_{\mathrm{m}}\right)$ that balances out ( $\mathrm{E}_{\mathrm{o}}$ ).
Hence, the net electric field inside the conductor becomes zero.

(ii) Dielectric


When external electric field is applied, dipoles are created (in case of non-polar dielectrics). The placement of dipoles is as shown in the given figure. An internal electric field is created which reduces the external electric field.
Polarization of dielectric ( P ) is defined as the dipole moment per unit volume of the polarized dielectric.
$\mathrm{P}=\chi_{\mathrm{e}} \varepsilon_{0} \mathrm{E}$
Where, $\chi_{\mathrm{e}}$ susceptibility
$\mathrm{E} \rightarrow$ Electric field
(b) Net force on the charge $\frac{Q}{2}$, placed at the centre of the shell, is zero.
Force on charge 2Q kept at a point A
$F=E \times 2 Q=\frac{I\left(\frac{3 Q}{2}\right) 2 Q}{4 \pi \varepsilon_{0} r^{2}}$
$F=\frac{k 3 Q^{2}}{r^{2}}$
Electric flux through the shell, $\phi=\frac{\mathrm{Q}}{\varepsilon_{0}}$
11. (a) Gaussian surface for a thin infinite plane sheet of uniform charge density

[1]
Let $\sigma$ be the surface charge density of the sheet. From symmetry, E on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet. We take a cylindrical cross-sectional area A and length 2 r as the Gaussian surface. On the curved surface of the cylinder E and $\hat{n}$ are perpendicular to each other. Therefore flux through curved surface $=0$. Flux through the flux surface $=\mathrm{EA}+\mathrm{EA}=2 \mathrm{EA}$
$\therefore$ Total electric flux over the centre surface of cylinder $\phi=2 \mathrm{EA}$
Total charge enclosed by the cylinder, $q=\sigma A$ acc. to Gauss' law, $\phi_{\mathrm{E}}=\frac{\mathrm{q}}{\varepsilon_{0}}$
$\therefore 2 \mathrm{EA}=\frac{\sigma \mathrm{A}}{\varepsilon_{0}}$
or $\mathrm{E}=\frac{\sigma}{2 \varepsilon_{0}}$
(b) Let $\mathrm{V}_{0}$ be the potential on the surface at sheet that at a distance $r$ from it
$d V=\vec{E} \cdot \overrightarrow{d r}$
$\mathrm{V}_{0}-\mathrm{V}=\frac{\sigma}{2 \varepsilon_{0}} \mathrm{r}$
$\mathrm{V}=\mathrm{V}_{0}-\frac{\sigma}{2 \varepsilon_{0}} \mathrm{r}$
12. (a) Electric flux is defined as, $\phi_{\varepsilon}=$ E. ds It is scalar quantity. Electric flux through
square is $\phi_{\varepsilon}=\frac{q}{\varepsilon_{0} 6}$
(b) Flux will not change, i.e. $\phi_{\varepsilon}=\frac{\mathrm{q}}{\varepsilon_{0} 6}$

## OR

To calculate the field, imagine a cylindrical Gaussian surface, as shown in the figure. Since the field is every where radial,flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, E is normal to the surface at every point,and its magnitude is constant, since it depends on $r$. The surface area of the curve if $2 \pi \mathrm{rl}$, where $l$ is the length of the cylinder.
[1]
Flux through the Gaussian surface
= flux through the curved cylindrical part of the surface

$$
=\mathrm{E} \times 2 \pi \mathrm{rl}
$$

The surface includes charge equal to $\lambda l$. Gauss's law then gives
$\mathrm{E} \times 2 \pi \mathrm{rl}=\frac{\lambda l}{\phi_{0}}$
i.e., $E=\frac{\lambda}{2 \pi r \phi_{0}}$

Vectorially, E at any point is given by $E=\frac{\lambda}{2 \pi r \phi_{0}} \hat{n}$

13.(a)

, Two gaussian spheres are $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$ For points $\mathrm{P}_{1}(\mathrm{x}<\mathrm{R})$ and $\mathrm{P}_{2}(\mathrm{x}>\mathrm{R})$. Now,
(i) By Gauss's law,

Net outward flux through $\mathrm{S}_{1}$

$$
\begin{align*}
\phi & =\oint_{\mathrm{s}_{1}} \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{dA}}=\frac{\mathrm{q}_{1}}{\varepsilon_{0}} \\
& \Rightarrow \mathrm{E}=0 \tag{1}
\end{align*}
$$

$\left[\because\right.$ change enclosed by $\mathrm{S}_{1}=0$ ]
(ii) Net outward flux through $\mathrm{S}_{2}$,

$$
\begin{aligned}
& \phi=\oint_{\mathrm{S}_{2}} \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{dA}}=\frac{\mathrm{q}_{2}}{\varepsilon_{0}} \\
& \Rightarrow \quad \mathrm{E} \oint_{\mathrm{s}_{2}} \mathrm{dA}=\frac{\sigma\left(4 \pi \mathrm{R}^{2}\right)}{\varepsilon_{0}} \\
& \text { Since } \oint \mathrm{dA}=4 \pi \mathrm{x}
\end{aligned}
$$

$\therefore \quad \mathrm{E}=\frac{\sigma(4 \pi \mathrm{R})^{2}}{\left(4 \pi \mathrm{x}^{2}\right) \varepsilon_{\mathrm{o}}}$
$\Rightarrow \quad \mathrm{E}=\frac{\sigma \mathrm{R}^{2}}{\varepsilon_{0} \mathrm{x}^{2}}$
(b)

(i) Net outward flux through cylinder :

$$
\begin{align*}
\phi & =\mathrm{f}_{1}+\mathrm{f}_{2}+\mathrm{f}_{3} \\
& =\mathrm{E}\left(\mathrm{pr}^{2}\right)+\mathrm{E}\left(\mathrm{pr}^{2}\right)+\mathrm{O} \\
& =2 \mathrm{E} \mathrm{pr} \\
& =2 \times 200 \times 3.14 \times\left(5 \times 10^{-2}\right)^{2} \\
& =400 \times 3.14 \times 25 \times 10^{-4} \\
& =3.14 \mathrm{~N}-\mathrm{m}^{2} / \mathrm{C} \tag{1}
\end{align*}
$$

(ii) Net charge, present inside the cylinder,

$$
\begin{align*}
& \mathrm{q}=\phi \mathrm{e}_{\mathrm{o}} \\
& \Rightarrow \quad \mathrm{q}=3.14 \times 8.854 \times 10^{-12} \\
& \quad=27.801 \times 10^{-12} \mathrm{C} \tag{1}
\end{align*}
$$

## MULTIPLE CHOICE QUESTIONS

1. A Cylinder of radius $R$ and length $L$ is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by
(a) $2 \pi R^{2} \mathrm{E}$
(b) $\frac{\pi \mathrm{R}^{2}}{\mathrm{E}}$
(c) $\frac{\left(\frac{\pi R^{2}}{\pi R}\right)}{\mathrm{E}}$
(d) zero
2. An electric charge $q$ is placed at the centre of a cube of side a. The electric flux on one of its faces will be
(a) $\frac{\mathrm{q}}{6 \varepsilon_{\mathrm{o}}}$
(b) $\frac{\mathrm{q}}{\varepsilon_{0} \mathrm{a}^{2}}$
(c) $\frac{\mathrm{q}}{4 \pi \varepsilon_{0} \mathrm{a}^{2}}$
(d) $\frac{\mathrm{q}}{\varepsilon_{\mathrm{o}}}$
3. Total electric flux coming out of a unit positive charge put in air is
(a) $\varepsilon_{0}$
(b) $\varepsilon_{0}^{-1}$
(c) $\left(4 \rho \varepsilon_{0}\right)^{-1}$
(d) $4 \pi \varepsilon_{0}$
4. For a given surface the Gauss's law is stated as $\oint E . d s=0$. From this we can conclude that
(a) E is necessarily zero on the surface
(b) E is perpendicular to the surface at every point
(c) The total flux through the surface is zero.
(d) The flux is only going out of the surface
5. A cube of side $\ell$ is placed in a uniform field E , where $\mathrm{E}=\mathrm{E} \hat{\mathrm{i}}$. The net electric flux through the cube is
(a) zero
(b) $\ell^{2} \mathrm{E}$
(c) $4 \ell^{2} \mathrm{E}$
(d) $6 \ell^{2} \mathrm{E}$
6. A charge $q$ is placed at the centre of the open end of cylindrical vessel. The flux of the electric field through the surface of the vessel is
(a) zero
(b) $\frac{q}{\varepsilon_{0}}$
(c) $\frac{\mathrm{q}}{2 \varepsilon_{\mathrm{o}}}$
(d) $\frac{2 q}{\varepsilon_{0}}$
7. According to Gauss's Theorem, electric field of an infinitely long straight wire is proportional to
(a) r
(b) $\frac{1}{\mathrm{r}^{2}}$
(c) $\frac{1}{\mathrm{r}^{3}}$
(d) $\frac{1}{\mathrm{r}}$
8. The S.I. unit of electric flux is
(a) Weber
(b) Newton percoulomb
(c) Volt $\times$ meter
(d) Joule per coulomb
9. Gauss's law is true only if force due to a charge varies as
(a) $\mathrm{r}^{-1}$
(b) $\mathrm{r}^{-2}$
(c) $\mathrm{r}^{-3}$
(d) $\mathrm{r}^{-4}$
10. An electric dipole is put in north-south direction in a sphere filled with water. Which statement is correct
(a) Electric flux is coming towards sphere
(b) Electric flux is coming out of sphere
(c) Electric flux entering into sphere and leaving the sphere are same
(d) Water does not permit electric flux to enter into sphere.

Answer Keys

1. (d)
2. (a)
3. (b)
4. (c)
5. (a)
6. (c)
7. (d)
8. (c)
9. (b)
10. (c)

## Solutions

1. Flux through surface $A \phi_{A}=E \times \pi R^{2}$ and $\phi_{B}=E$ $\times \pi \mathrm{R}^{2}$.


Flux through curved surface
$\mathrm{C}=\int \overrightarrow{\mathrm{E}} \cdot \mathrm{d} \overrightarrow{\mathrm{s}}=\int \mathrm{E} \mathrm{ds} \cos 90^{\circ}=0$
$\therefore$ Total flux through cylinder $=\phi_{\mathrm{A}}+\phi_{\mathrm{B}}+\phi_{\mathrm{C}}=0$
2. By Gauss's theorem,

Electric flux $(\phi)=\frac{q}{6 \varepsilon_{0}}$
3. Total flux coming out from unit charge
$\phi=\overrightarrow{\mathrm{E}} \cdot \mathrm{d} \overrightarrow{\mathrm{s}}=\frac{1}{\varepsilon_{0}} \times 1=\varepsilon_{\mathrm{o}}{ }^{-1}$
4. The total flux through the surface is zero.
5. As there is no charge residing inside the cube, hence net flux is zero.
[1]
6. To apply Gauss's theorem it is essential that charge should be placed inside a closed surface. So, imagine another similar cylindrical vessel above it as shown in figure (dotted).
$\therefore$ Required flux $\phi=\frac{\mathrm{q}}{2 \varepsilon_{\mathrm{o}}}$

7. $\mathrm{E}=\frac{\lambda}{2 \pi \varepsilon_{0} \mathrm{r}} \Rightarrow \mathrm{E} \propto \frac{1}{\mathrm{r}}$
8. S.I. unit of electric flux is
$\frac{\mathrm{N} \times \mathrm{m}^{2}}{\mathrm{C}}=\frac{\mathrm{J} \times \mathrm{m}}{\mathrm{C}}=$ volt $\times$ metre
9. Gauss's law is true only if force due to a charge varies as $\mathrm{r}^{-2}$.
10. In electric dipole the flux coming out from positive charge is equal to the flux coming in at negative charge i.e. total charge on sphere $=0$. From Gauss law, total flux passing through the sphere $=0$.

## CHAPTER 2

## Electrostatic Potential and Capacitance

## Topic 1: Electrostatic Potential and Electrostatic Potential Energy

## Summary

## Electrostatic potential:

- The amount of work done by an external force in moving a unit positive charge from one point to another in electrostatic field is called electrical potential.
- Such that $\mathrm{V}=\frac{1}{4 \pi \varepsilon} \frac{\mathrm{q}}{\mathrm{r}}$
- Where, $\mathrm{q}=$ charge causing the field, $\varepsilon=$ permittivity, $r=$ separation between centre of charge point.
- Electrostatic force is a conservative force.
- Work done by an external force (equal and opposite to the electrostatic force) in bringing a
charge $q$ from a point $R$ to a point $P$ is $V_{P}-V_{R}$, which is the difference in potential energy of charge $q$ between the final and initial points.


## Potential difference:

When the work is done upon a charge to change its potential energy then the difference between the final and the initial location is called electric potential difference.

## Electric Potential due to a dipole:

- The electrostatic potential at a point with distance $r$ due dipole at a point making an angle $\theta$ with dipole moment $p$ placed at the origin is given by $\mathrm{V}(\mathrm{r})=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \cdot \frac{\mathrm{p} \cdot \hat{\mathrm{r}}}{\mathrm{r}^{2}}$.


Fig. Electrical potential due to dipole

- It is a scalar quantity.
- Let A and B be the initial and final location for a single charge $q$ then the potential difference between $A$ and $B$ is given by:
$\Delta \mathrm{V}=\mathrm{V}_{\mathrm{B}}-\mathrm{V}_{\mathrm{A}}=-\int_{\mathrm{A}}^{\mathrm{B}} \mathrm{E} \times \mathrm{ds}=-\int_{\mathrm{A}}^{\mathrm{B}} \mathrm{Eds} \cos \theta=-\int_{\mathrm{A}}^{\mathrm{B}} \mathrm{E} \times \mathrm{ds}$
Where, E is the field due to a point charge, $\mathrm{ds}=\mathrm{dr}$, so that
$\mathrm{V}_{\mathrm{B}}-\mathrm{V}_{\mathrm{A}}=\int_{\mathrm{r}_{\mathrm{A}}}^{\mathrm{r}_{\mathrm{B}}} \frac{\mathrm{q}}{4 \pi \varepsilon_{0}} \frac{\mathrm{dr}}{\mathrm{r}^{2}}=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{1}{\mathrm{r}}\right]_{\mathrm{r}_{\mathrm{A}}}^{\mathrm{r}_{\mathrm{B}}}=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{1}{\mathrm{r}_{\mathrm{B}}}-\frac{1}{\mathrm{r}_{\mathrm{A}}}\right]$
- The result is true also for a dipole (with charges $-q$ and $q$ separated by 2 a for $r \gg a$.


## Dipole and System of charges

- For a charge configuration $q_{1}, q_{2}, \ldots \ldots, q_{n}$ with position vectors $r_{1}, r_{2}, r_{3}, \ldots \ldots, r_{n}$, then the potential $V_{1}$ at point $P$ due to charge $q_{1}$ will be,

$$
\mathrm{V}_{1}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1}}{\mathrm{r}_{1}}
$$

And the sum of potentials due to individual charges is given by the superposition principle,
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}}\left(\frac{\mathrm{q}_{1}}{\mathrm{r}_{1 \mathrm{P}}}+\frac{\mathrm{q}_{2}}{\mathrm{r}_{2 \mathrm{P}}}+\ldots .+\frac{\mathrm{q}_{\mathrm{n}}}{\mathrm{r}_{\mathrm{nP}}}\right)$


- In a uniformly charged spherical shell, the electric field outside the shell with outside potential is given by,

$$
\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}}
$$

## Equipotential surfaces

- A surface over which potential has a constant value is known as an equipotential surface.
- The amount of work done in moving a charge over an equipotential surface is zero.
- Concentric spheres centered at a location of the charge act as equipotential surfaces for a point charge.
- The electric field E, at a point and equipotential surface are mutually perpendicular to each other through the point. The direction of the steepest decrease of potential is in $E$.
- Regionsofstrongandweakfieldsarelocatedbecause of the spacing among equipotential surfaces.


## Potential Energy of a System of Charges:

Potential energy stored in a system of charges is the work done by an external agency in assembling the charges at their locations. Total work done in assembling the charges is given by $\mathrm{U}=\frac{1}{4 \pi \varepsilon_{0}} \cdot\left(\frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}+\frac{\mathrm{q}_{1} \mathrm{q}_{3}}{\mathrm{r}_{13}}+\frac{\mathrm{q}_{2} \mathrm{q}_{3}}{\mathrm{r}_{23}}\right)$ where $\mathrm{r}_{12}$ is distance between $q_{1}$ and $q_{2}, r_{13}$ is distance between $q_{1} \& q_{3}$ and $\mathrm{r}_{23}$ is distance between $\mathrm{q}_{2} \&$ relabel $\mathrm{q}_{3}$.


Fig. Potential energy due to System of charges

## Electric potential energy of system of two point charges

- Here the work done doesn't depend on path.
- In this system the two charges $q_{1}$ and $q_{2}$ when separated by distance $r$, will either repel or attract each other.
- Electrical potential of charges $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ is given by: $\mathrm{U}=\frac{1}{2} \sum_{\mathrm{i}=1}^{2} \mathrm{q}_{\mathrm{i}} \mathrm{V}_{\mathrm{i}}$


## Potential Energy in an External Field:

- The potential energy of a charge q in an external potential $\mathrm{V}(\mathrm{r})$ is $\mathrm{qV}(\mathrm{r})$. The potential energy of a dipole moment p in a uniform electric field E is -p.E.
- Electric dipole in an electrostatic field: Electric potential due to a dipole at a point at distance $r$ and making an angle $\theta$ with the dipole moment $p$ is given by
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{p} \cos \theta}{\mathrm{r}^{2}}$


## Electrostatics of conductors:

- Electrostatic field is zero inside a conductor.
- Electrostatic field at the surface of a charged conductor must be normal to the surface at every point.
- In the static situation, there cannot be any excess charge in the interior of a conductor.
- Throughout the volume of the conductor, the electrostatics potential is constant and has same value on its surface.
- Electrostatics field $E$ is zero in the interior of a conductor; just outside the surface of a charged conductor, E is normal to the surface given by $\mathrm{E}=\frac{\sigma}{\varepsilon_{0}} \hat{\mathrm{n}}$ where $\hat{\mathrm{n}}$ is the unit vector along the outward normal to the surface and $\sigma$ is the surface charge density.
- Electrostatic shielding: A field which is inside the cavity of a conductor is always zero and it remains shielded from the electric field, which is known as electrostatic shielding.


## Dielectrics and Polarization:

- Dielectrics: A non-conducting substance which has a negligible number of charge carriers unlike conductors is called dielectrics.
- Electric polarization: The difference between induced electric field and imposed electric field in dielectric due to bound and free charges is known as electric polarization. It is written as:
$P=\frac{D-E}{4 \pi}$
Note: Polarisation can also be written as polarization (with ' $z$ ' in place of ' $s$ ')


## PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 1

## ■ 1 Mark Questions

1. A point charge $Q$ is placed at point $O$ as shown in the figure. Is the potential difference $V_{A}-V_{B}$ positive, negative or zero, if $Q$ is (i) positive (ii) negative?

[ALL INDIA 2011]
2. For any charge configuration, equipotential surface through a point is normal to the electric field. Justify.
[DELHI 2014]
3. What is the amount of work done in moving a point charge $Q$ around a circular arc of radius ' $r$ ' at the centre of which another point charge ' $q$ ' is located?
[ALL INDIA 2016]
4. The electric potential V at any point ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) is given by $V=3 x^{2}$ where $x$ is in metres and $V$ in volts. The electric field at the point ( $1 \mathrm{~m}, 0,2 \mathrm{~m}$ ) is -
(a) $6 \mathrm{~V} / \mathrm{m}$ along - x -axis
(b) $6 \mathrm{~V} / \mathrm{m}$ along + x-axis
(c) $1.5 \mathrm{~V} / \mathrm{m}$ along -x -axis
(d) $1.5 \mathrm{~V} / \mathrm{m}$ along +x -axis
[DELHI TERM I, 2022]
5. Four objects $\mathrm{W}, \mathrm{X}, \mathrm{Y}$ and Z , each with charge $+q$ are held fixed at four points of a square of side d as shown in the figure. Objects X and Z are on the midpoints of the sides of the square. The electrostatic force exerted by object W on object X is F . Then the magnitude of the force exerted by object W on Z is

(a) $\frac{\mathrm{F}}{7}$
(b) $\frac{\mathrm{F}}{5}$
(c) $\frac{\mathrm{F}}{3}$
(d) $\frac{\mathrm{F}}{2}$
[DELHI TERM I, 2022]
6. Two charges $14 \mu \mathrm{C}$ and $-4 \mu \mathrm{C}$ are placed at ( -12 cm , $0,0)$ and ( $12 \mathrm{~cm}, 0,0$ ) in an external electric field $\mathrm{E}=\left(\frac{\mathrm{B}}{\mathrm{r}^{2}}\right)$, where $\mathrm{B}=1.2 \times 10^{6} \mathrm{~N} /\left(\mathrm{cm}^{2}\right)$ and r is in metres. The electrostatic potential energy of the configuration is
(a) 97.9 J
(b) 102.1 J
(c) 2.1 J
(d) -97.9 J
[DELHI TERM I, 2022]
7. Equipotentials at a large distance from a collection of charges whose total sum is not zero are-
(a) spheres
(b) planes
(c) ellipsoids
(d) paraboloids
[DELHI TERM I, 2022]
8. Four charges $-q,-q,+q$ and $+q$ are placed at the corners of a square of side 2 L is shown in figure. The electric potential at point A midway between the two charges $+q$ and $+q$ is -

(a) $\frac{1}{4 \pi \epsilon_{0}} \frac{2 q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$
(b) $\frac{1}{4 \pi \epsilon_{0}} \frac{2 \mathrm{q}}{\mathrm{L}}\left(1+\frac{1}{\sqrt{5}}\right)$
(c) $\frac{1}{4 \pi \in_{0}} \frac{\mathrm{q}}{2 \mathrm{~L}}\left(1-\frac{1}{\sqrt{5}}\right)$
(d) zero
[DELHI TERM I, 2022]

## ■ 2 Marks Questions

9. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the X-Z plane at a distance ' $d$ ' apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass $m$ and charge ' $-q$ ' remains stationary between the plates, what is the magnitude and direction of the field?

Or
Two small identical electrical diploes $A B$ and $C D$, each of dipole moment ' p ' are kept an angle of $120^{\circ}$ as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field $E$ directed along $+X$ direction, what will be the magnitude and direction of the torque acting on this?
[ALL INDIA 2011]
10. An electric dipole of length 2 cm , when placed with its axis making an angle of $60^{\circ}$ with a uniform electric field, experiences a torque of $6 \sqrt{ } 3 \mathrm{Nm}$. Calculate the potential energy of the dipole, if it has a charge of $\pm 2 n C$.
[DELHI 2014]
11. An electric dipole of length 2 cm , when placed with its axis making an angle of $60^{\circ}$ with a uniform electric field, experiences a torque of $8 \sqrt{ } 3 \mathrm{Nm}$. Calculate the potential energy of the dipole, if it has a charge of $\pm 4 n C$.
[DELHI 2014]
12. (i) Can two equipotential surfaces intersect each other? Give reasons.
(ii) Two charges $-q$ and $+q$ are located at points $A(0,0,-\mathrm{a})$ and $B(0,0,+\mathrm{a})$ respectively. How much work is done in moving a test charge from point $P(97,0,0)$ to $Q(-3,0,0)$ ?
[DELHI 2017]
13. Three-point charges $Q, q$ and $-q$ are kept at the vertices of an equilateral triangle of side $L$ as shown in figure. What is

(i) The electrostatic potential energy of the arrangement? and
(ii) The potential at point D? [DELHI 2023]

■ 3 Marks Questions
14. (a) Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.
(b) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.
[ALL INDIA 2017]
15. (i) Draw equipotential surfaces for a system of two identical positive point charges placed a distance ' $d$ ' apart.
(ii) Deduce the expression for the potential energy of a system of two point charges $q_{1}$ and $q_{2}$ brought from infinity to the points $\overrightarrow{r_{1}}$ and $\overrightarrow{r_{2}}$ respectively in the presence of external electric field $\vec{E}$. [DELHI 2017]
16. Four point charges $Q, q, Q$ and $q$ are place $d$ at the corners of a square of side ' $\alpha$ ' as shown in the figure.


Find the (a) resultant electric force on a charge $Q$, and (b) Potential energy of the system

OR
(a) Three point charge $s q,-4 q$ and $2 q$ are place $d$ at the vertices of an equilateral triangle $A B C$ of side ' $l$ ' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge $q$.

(b) Find out the amount of the work done to separate the charges a t infinite distance.
[ALL INDIA 2018]

## ■ 5 Marks Questions

17. (a) State Gauss's law in electrostatics. Show, with the help of a suitable example along with the figure, that the outward flux due to a point charge ' $q$ '. in vacuum within a closed surface, is independent of its size or shape and is given by $\frac{q}{\varepsilon_{0}}$
(b) Two parallel uniformly charged infinite plane sheets, ' 1 ' and ' 2 ', have charge densities $+\sigma$ and $-2 \sigma$ respectively. Give the magnitude and direction of the net electric field at a point.
(i) in between the two sheets and
(ii) outside near the sheet ' 1 '.

Or
(a) Define electrostatic potential at a point. Write its S.I. unit. Three point charges $q_{1}$, $q_{2}$ and $q_{3}$ are kept respectively at points A , $B$ and $C$ as shown in the figure, Derive the expression for the electrostatic potential energy of the system.

(b) Depict the equipotential surface due to (i) an electric dipole, (ii) two identical positive charges separated by a distance.
[DELHI 2015]
18. (a) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point. Draw a sketch of equipotential surfaces due to a single charge $(-q)$, depicting the electric field lines due to the charge
(b) Obtained an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.

[ALL INDIA 2016]
19. (a) Find the expression for the potential energy of a system of two point charges $q_{1}$ and $q_{2}$ located at ${ }^{\rightarrow}$ and $\overrightarrow{\mathrm{r}}_{2}$, respectively in an external electric field $\overrightarrow{\mathrm{E}}$.
(b) Draw equipotential surfaces due to an isolated point charge ( -q ) and depict the electric field lines.
(c) Three point charges $+1 \mu \mathrm{C},-1 \mu \mathrm{C}$ and $+2 \mu \mathrm{C}$ are initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10 cm .
[DELHI 2020]

## Solutions

1. Potential at a point: $\mathrm{V}=\frac{\mathrm{kq}}{\mathrm{r}}$, Given $q=Q$

$$
\mathrm{V}_{\mathrm{A}}-\mathrm{V}_{\mathrm{B}}=\mathrm{kQ}\left[\frac{1}{\mathrm{r}_{\mathrm{A}}}-\frac{1}{\mathrm{r}_{\mathrm{B}}}\right]^{1}
$$



Where, $r_{\mathrm{A}}<r_{\mathrm{B}}$
So, $\frac{1}{\mathrm{r}_{\mathrm{A}}}>\frac{1}{\mathrm{r}_{\mathrm{B}}}$
$\therefore\left[\frac{1}{\mathrm{r}_{\mathrm{A}}}-\frac{1}{\mathrm{r}_{\mathrm{B}}}\right]>0$
If $Q$ at $O$ is positive, $V_{\mathrm{A}}-V_{\mathrm{B}}$ will be positive [1/2] If $Q$ at $O$ is negative, $V_{\mathrm{A}}-V_{\mathrm{B}}$ will be negative.
2. If the electric field were not normal to equipotential surface, it would have non-zero component along the surface. To move a charge against this component, work would have to be done. But no work is needed to move a test charge on an equipotential surface. Hence electric field must be normal to the equipotential surface at every point.
[1]
3. Given charge ' $q$ ' is located at centre and charge ' $Q$ ' on surface. Then, work done

$$
\begin{equation*}
\mathrm{W}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \frac{\mathrm{q} \cdot \mathrm{Q}}{\mathrm{r}^{2}} \tag{1/2}
\end{equation*}
$$

Work done to move the charge over the circular arc is zero, because it is moving over an equipotential surface.
[1/2]
4. (a) Electric potential $V=3 x^{2}$

$$
\begin{aligned}
& E=-\frac{d V}{d x} \\
& E=-\frac{d\left(3 x^{3}\right)}{d x}=-6 x
\end{aligned}
$$

At the point (1,0,2)
Electric field $\mathrm{E}=6 \times 1=-6 \mathrm{~V} / \mathrm{m}$
Thus electric field at $(1 \mathrm{~m}, 0,2 \mathrm{~m})$ is
$6 \mathrm{Vm}^{-1}$ along negative x -axis.
5. (b) $\frac{\mathrm{F}}{5}$

Force on X by W is $\mathrm{F}=\frac{\mathrm{kq}^{2}}{\left(\frac{\mathrm{~d}}{\mathrm{z}}\right)^{2}}$


$$
\mathrm{WZ}=\sqrt{\mathrm{d}^{2}+\left(\frac{\mathrm{d}}{2}\right)^{2}}=\sqrt{\frac{5 \mathrm{~d}^{2}}{4}}
$$

Force on Z by W is

$$
\begin{equation*}
\mathrm{F}^{\prime}=\frac{\mathrm{kq}^{2}}{\frac{5 \mathrm{~d}^{2}}{4}}=\frac{4 \mathrm{kq}^{2}}{5 \mathrm{~d}^{2}}=\frac{\mathrm{F}}{\mathrm{~S}} \tag{1}
\end{equation*}
$$

6. (a) 97.9 J

We know that, Energy
$-q_{1} V+q_{2} V+\frac{\mathrm{kq}_{1} q_{2}}{r}$
$\mathrm{E}=\frac{\mathrm{B}}{\mathrm{r}_{2}}$
So, $V=-\int \frac{B}{r^{3}} \mathrm{dr}=\frac{\mathrm{B}}{4}=1.2 \times 10^{6} / \mathrm{r}$
Now, Energy

$$
\begin{gather*}
=\frac{14 \times 10^{-6} \times 1.2 \times 10^{6}}{\left(12 \times 10^{-2}\right)}-\frac{14 \times 10^{-6} \times 1.2 \times 10^{6}}{\left(12 \times 10^{-2}\right)} \\
-\frac{9 \times 10^{9} \times 14 \times 10^{-6} \times 4 \times 10^{6}}{\left(24 \times 10^{-2}\right)} \\
=(14-4) \times 1.2\left(12 \times 10^{-2}\right)-\frac{9 \times 56}{240} \\
=100-2.1=97.9 \mathrm{~J} \tag{1}
\end{gather*}
$$

7. (a) A collection of charge located at a very large distance can be considered as the point charge. Now the equipotential surface for a point charge will have the same distance from the point in all the directions.
Therefore, the equipotential points for a point charge will have a spherical surface. So, Option A is correct.
8. (a) $\frac{1}{4 \pi \epsilon_{0}} \times \frac{2 q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$


Electric potential due to two $+q$ charges $=\frac{1}{4 \pi \epsilon_{0}} \times \frac{2 q}{L}$

Electric potential due to two -q charges $=\frac{1}{4 \pi \epsilon_{0}} \times \frac{-2 q}{\sqrt{5} \mathrm{~L}}$
Total Potential at

$$
\begin{equation*}
\mathrm{A}=\frac{1}{4 \pi \epsilon_{0}} \times \frac{-2 \mathrm{q}}{\mathrm{~L}}\left(1-\frac{1}{\sqrt{5}}\right) \tag{1}
\end{equation*}
$$

9. 



Fig. Two uniformly charged plates
Here the dotted lines represent the parallel equipotential surfaces along X-Z plane.
If a charge $q$ has to be held stationary between the two plates, it will have to be balanced by two forces.
Gravitational force: mg, downwards
Electrostatic force $=2 q E$, acting upwards.
This implies, that in X-Z plane, the upper plate is + charged plate \& lower plate is -charged plate.
So, E field lines have to be directed along - y axis.

[1/2]
Fig. An Electric dipole
Resultant dipole moment,

$$
\begin{aligned}
& \overrightarrow{\mathrm{P}}_{\mathrm{res}}=\overrightarrow{\mathrm{p}}_{1}+\overrightarrow{\mathrm{p}}_{2} \\
& \mathrm{p}_{\mathrm{res}}=\sqrt{\mathrm{p}_{1}^{2}+\mathrm{p}_{2}^{2}+2 \mathrm{p}_{1} \mathrm{p}_{2} \cos 120^{\circ}} \\
& P_{\mathrm{res}}=p
\end{aligned}
$$

Direction of resultant dipole moment:
$\tan \theta=\frac{\mathrm{p} \sin 120^{\circ}}{\mathrm{p}+\mathrm{p} \cos 120^{\circ}}, \tan \theta=\sqrt{3}, \theta=60^{\circ}$
That is, 30 degrees with +x axis.
Given applied $E$ is along +x axis,
So torque on resultant dipole will be
$\tau=\mathrm{pE} \sin 30^{\circ}=\frac{\mathrm{pE}}{2}$
Direction will be along-Z-axis.
10. As $\tau=P E \sin \theta$
$6 \sqrt{3}=p E \sin 60^{\circ}=p E \times \frac{\sqrt{3}}{2}$ or $P E=12$
$U=-P E \cos \theta$
Potential energy of a dipole

$$
\begin{equation*}
=-12 \cos 60^{\circ}=-12 \times \frac{1}{2}=-6 J \tag{1}
\end{equation*}
$$

11. As $\tau=P E \sin \theta$
$8 \sqrt{3}=p E \sin \theta$
$8 \sqrt{3}=p E \sin 60^{\circ}$
$8 \sqrt{3}=\frac{p E \sqrt{3}}{2}$
$\Rightarrow p E=16$
Also Potential energy of the dipole,
$\Rightarrow U=-P E \cos \theta$
$U=-P E \cos 60^{\circ}$
$U=\frac{-16.1}{2}=-8 J$
$U=-8 J$
12. (i) Two equipotential surfaces cannot intersect each other because when they will intersect, the electric field will have two directions, which is impossible.
[1]
(ii) Charge $P$ moves on the perpendicular bisector of the line joining $+q$ and $-q$. Hence, this perpendicular bisector is equidistant from both the charges. Thus, the potential will be same everywhere on this line. Therefore, work done will be zero.
13. 


(i) Electrostatic potential energy of system

$$
\begin{align*}
& =\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{Q}(-\mathrm{q})}{\mathrm{L}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-\mathrm{q})(\mathrm{q})}{\mathrm{L}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{Q} \times \mathrm{q}}{\mathrm{~L}} \\
& \Rightarrow \frac{-\mathrm{Qq}}{4 \pi \varepsilon_{0} \mathrm{~L}}+\frac{-\mathrm{q}^{2}}{4 \pi \varepsilon_{0} \mathrm{~L}}+\frac{\mathrm{Qq}}{4 \pi \varepsilon_{0} \mathrm{~L}}=\frac{-\mathrm{q}^{2}}{4 \pi \varepsilon_{0} \mathrm{~L}} \tag{1}
\end{align*}
$$

(ii) Potential at point, D
$=\frac{K \times q}{\frac{L}{2}}+\frac{(-K q)}{\frac{L}{2}}+\frac{K \times Q}{\frac{\sqrt{3} L}{2}}$
$=\frac{1}{4 \pi \varepsilon_{0}} \times \frac{2 \mathrm{Q}}{\sqrt{3} \mathrm{~L}}=\frac{\mathrm{Q}}{2 \sqrt{3} \pi \varepsilon_{0} \mathrm{~L}}$
14. (a) Electric dipole moment of an electric dipole is defined as the product of the magnitude of either charge of the electric dipole and the dipole length.


Fig. An Electric Dipole
i.e., $\vec{p}=q(2 \vec{l})$

The magnitude of dipole moment is $p=q \times 2 l$ Dipole moment is a vector quantity. SI Unit of dipole moment $(\vec{p})$ is coulomb metre $(\mathrm{Cm})$


Fig: Electrical field of a dipole at a point on equatorial plane of the dipole
Electric field intensity due to $+q$ charge is given by

$$
\begin{equation*}
E_{+}=\frac{1}{4 \pi \varepsilon_{o}} \frac{q}{B P^{2}} \tag{i}
\end{equation*}
$$

Electric field intensity due to $-q$ charge is given by

$$
\begin{equation*}
E_{-}=\frac{1}{4 \pi \varepsilon_{o}} \frac{q}{A P^{2}}=\frac{1}{4 \pi \varepsilon_{o}} \frac{q}{\left(r^{2}+l^{2}\right)} \tag{ii}
\end{equation*}
$$

From (i) and (ii),
$E_{+}=E_{-}=\frac{1}{4 \pi \varepsilon_{o}} \frac{q}{\left(r^{2}+l^{2}\right)}$
[1/2]

The net electric field intensity due to the electric dipole.
$\therefore E=\sqrt{E_{+}^{2}+E_{-}^{2}+2 E_{+} E_{-} \cos 2 \theta}$
$=\sqrt{E_{+}^{2}+E_{+}^{2}+2 E_{+}^{2} \cos 2 \theta}\left(E_{+}=E_{-}\right)$
$=\sqrt{2 E_{+}^{2}+2 E_{+}^{2} \cos 2 \theta}$
$=\sqrt{2 E_{+}^{2}(1+\cos 2 \theta)}$
$=\sqrt{2 E_{+}^{2} \times 2 \cos ^{2} \theta}\left(1+\cos 2 \theta=2 \cos ^{2} \theta\right)$
$\therefore E=2 E_{+} \cos 2 \theta=2 \times \frac{1}{4 \pi \varepsilon_{o}} \frac{q}{\left(r^{2}+l^{2}\right)} \cos \theta$
[1/2]
[Using equation (iii)]
$\cos \theta=\frac{l}{\sqrt{\left(r^{2}+l^{2}\right)}}$
$E=\frac{q}{4 \pi \varepsilon_{o}\left(r^{2}+l^{2}\right)^{\left(\frac{3}{2}\right)}}$
If l $\ll r, E=\frac{q}{4 \pi \varepsilon_{0} r^{3}}$
(b)


Fig. Equipotential Surface due to dipole
Electrical potential is zero at all points in the plane passing through the dipole equator.
15. (i)

[1]

Fig. Equipotential Surface for System of Charges
(ii) The work done in bringing charge q 1 from infinity to $\overrightarrow{r_{1}}$ is $q_{1} V\left(\overrightarrow{r_{1}}\right)$.
Work done on $q 2$ against external field
$=q_{2} V\left(\overrightarrow{r_{2}}\right)$
Work done on q2 against the field due to
$q_{1}=\frac{q_{1} q_{2}}{4 \pi \varepsilon_{o} r_{12}}$
Where, $r_{12}$ is the distance between $q_{1}$ and $q_{2}$. By the superposition principle for fields,
Work done in bringing $q_{2}$ to
$\overrightarrow{r_{2}}$ is $\left(q_{2} V\left(\overrightarrow{r_{2}}\right)+\frac{q_{1} q_{2}}{4 \pi \varepsilon_{o} r_{12}}\right)$
Thus,
Potential energy of system = The total work done in assembling the configuration

$$
\begin{equation*}
=q_{1} V\left(\overrightarrow{r_{1}}\right)+q_{2} V\left(\overrightarrow{r_{2}}\right)+\frac{q_{1} q_{2}}{4 \pi \varepsilon_{o} r_{12}} \tag{1}
\end{equation*}
$$

16. (a) There will be three forces on charge $Q$

$F_{1}=\frac{1}{4 \pi \varepsilon_{o}} \frac{q Q}{a^{2}}$
$F_{2}=\frac{1}{4 \pi \varepsilon_{o}} \frac{q Q}{a^{2}}$
$F_{3}=\frac{1}{4 \pi \varepsilon_{o}} \frac{Q Q}{(\sqrt{2} a)^{2}}=\frac{1}{4 \pi \varepsilon_{o}} \frac{Q^{2}}{2 a^{2}}$
$F_{1}$ and $F_{2}$ are perpendicular to each other so their resultant will be
$F^{\prime}=\sqrt{F_{1}^{2}+F_{2}^{2}+2 F_{1} F_{2} \cos 90^{\circ}}$
$F_{1}=F_{2}$
$F^{\prime}=\sqrt{F_{1}^{2}+F_{1}^{2}+2 F_{1} F_{1} \times 0}$
$F^{\prime}=\sqrt{2 F_{1}^{2}}=\sqrt{2} F_{1}$
$F_{3}$ and resultant of $F_{1}$ and $F_{2}$ will be in same direction
Net force $F=F^{\prime}+F_{3}$

$$
\begin{align*}
& F=\sqrt{2} \frac{1}{4 \pi \varepsilon_{o}} \frac{q Q}{a^{2}}+\frac{1}{4 \pi \varepsilon_{o}} \frac{Q^{2}}{2 a^{2}} \\
& F=\frac{1}{4 \pi \varepsilon_{o}} \frac{q Q}{a^{2}}\left(q \sqrt{2}+\frac{Q}{2}\right) \tag{1}
\end{align*}
$$

(b)

$w=\frac{4 K Q q}{a}+\frac{K q^{2}}{\sqrt{2} a}+\frac{K Q^{2}}{\sqrt{2} a}$
$K=\frac{1}{4 \pi \varepsilon_{o}}$
[1/2]

$\left|\overrightarrow{F_{1}}\right|=\frac{1}{4 \pi \varepsilon_{o}} \frac{(4 q)(q)}{l^{2}}$
$F_{1}=\frac{1}{4 \pi \varepsilon_{o}} \frac{4 q^{2}}{l^{2}}$
$F_{1}=\frac{1}{\pi \varepsilon_{o}} \frac{q^{2}}{l^{2}}$
$\left|\overrightarrow{F_{2}}\right|=\frac{1}{4 \pi \varepsilon_{o}} \frac{(q)(2 q)}{l^{2}}$
$F_{2}=\frac{1}{2 \pi \varepsilon_{o}} \frac{q^{2}}{l^{2}}$

Angle between $F_{1}$ and $F_{12}$ is $120^{\circ}$
$\vec{F}=\sqrt{F_{1}^{2}+F_{2}^{2}+2 F_{1} F_{2} \cos 120^{\circ}}$
$F_{1}=2 F_{2}$
$F=\sqrt{\left(2 F_{2}\right)^{2}+F_{2}^{2}+4 F_{2}^{2} \cos 120^{\circ}}$
$F=\sqrt{4 F_{2}^{2}+F_{2}^{2}-2 F_{2}^{2}}$
$F=\sqrt{3 F_{2}^{2}}$
$F=\sqrt{3} F_{2}$
$F=\frac{\sqrt{3}}{2 \pi \varepsilon_{o}} \frac{q^{2}}{l^{2}}$
(b) The amount of work done to separate the charges at infinity will be equal to potential energy.
$U=\frac{1}{4 \pi \varepsilon_{o} l}[q \times(-4 q)+(q \times 2 q)+(-4 q \times 2 q)]$
$U=\frac{1}{4 \pi \varepsilon_{o} l}\left[-4 q^{2}+2 q^{2}-8 q^{2}\right]$
$U=\frac{1}{4 \pi \varepsilon_{o} l}\left[-10 q^{2}\right]$
$U=-\frac{1}{4 \pi \varepsilon_{o} l}\left[10 q^{2}\right]$ unit
17. (a) Statement: The electric flux linked with a closed surface is equal to $\varepsilon$ times the net charge enclosed by a closed surface.
Mathematical expression:

$$
\begin{equation*}
\phi_{E}=\oint E \cdot d s=\frac{1}{\varepsilon_{o}}\left(q_{n e t}\right) \tag{1/2}
\end{equation*}
$$

Consider two spherical surfaces of radius $r$ and $2 r$ respectively and a charge 1 is enclosed in it. According to gauss theorem the total electric flux linked with a closed surface depends on the charge enclosed in it so For fig (a)

(b)

$$
\begin{equation*}
\phi_{E}=\frac{q}{\varepsilon_{o}} \tag{1/2}
\end{equation*}
$$


(a)

And for fig (b)
$\phi_{E}=\frac{q}{\varepsilon_{o}}$ which is same in both the cases so it is independent of size and shape of closed surface.
(b)

[1/2]
b)
[1/2]

Let $\hat{r}$ be the unit vector directed from left to right
Let $P$ and $Q$ are two points in the inner and outer region of two plates respectively charge densities on plates are $+\sigma$ and $-2 \sigma$
[1/2]
(i) Electric field at point P in the inner region of the plates
$E=E_{1} E_{2}$
$E=\left(\frac{\sigma}{2 \varepsilon_{o}}+\frac{\sigma}{\varepsilon_{o}}\right) r$
$E=\frac{3 \sigma}{2 \varepsilon_{o}} r$
(ii) Electric field at point $Q$ in the outer region of plate 1
$E_{1}=\frac{\sigma}{2 \varepsilon_{o}}(-r)$ and $E_{2}=\frac{2 \sigma}{2 \varepsilon_{o}} r$
$\therefore \quad$ Net electric field in the outer region of the plate 1 (i.e, at $Q$ ) is

$$
\begin{align*}
& E=E_{1}+E_{2}=\left(\frac{\sigma}{\varepsilon_{o}}-\frac{\sigma}{2 \varepsilon_{o}}\right) r  \tag{1/2}\\
& E=\frac{\sigma}{2 \varepsilon_{o}} r \tag{1/2}
\end{align*}
$$

Or
(a) The electrostatic potential ( $V$ ) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. Its S.I. unit is Volt. The potential energy of a system of three charges $q_{1}, q_{2}$ and $q_{3}$ located at $\mathrm{r}_{1}, r_{2}$ and $r_{3}$ respectively. To bring $q_{1}$ first from infinity to $\mathrm{r}_{1}$ no work is required. Next we bring $q_{2}$ from infinity to $r_{2}$. Work done is

$$
\begin{equation*}
q_{2} V_{1}\left(r_{2}\right)=\frac{1}{4 \pi \varepsilon_{o}} \frac{q_{1} q_{2}}{r_{12}} \tag{1}
\end{equation*}
$$

The charge $q_{1}$ and $q_{2}$ produce at potential, which at any point $P$ is given by
$V_{1 \cdot 2}=\frac{1}{4 \pi \varepsilon_{o}}\left(\frac{q_{1}}{r_{1 P}}+\frac{q_{2}}{r_{2 P}}\right)$
Work done next in bringing $q_{3}$ from infinity to the point $r_{3}$ is $q_{3}$ times $V_{12}$ at

$$
\begin{equation*}
q_{3} V_{1,2}\left(r_{3}\right)=\frac{1}{4 \pi \varepsilon_{o}}\left(\frac{q_{1} q_{2}}{r_{12}}+\frac{q_{2} q_{3}}{r_{23}}\right) \ldots \tag{3}
\end{equation*}
$$

The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps [Eq. (1) and Eq. (3)] and gets stored in the form of potential energy.
$U=\frac{1}{4 \pi \varepsilon_{o}}\left(\frac{q_{1} q_{2}}{r_{12}}+\frac{q_{1} q_{3}}{r_{13}}+\frac{q_{2} q_{3}}{r_{23}}\right)$
(b) Equipotential surfaces for (a) a dipole and (b) two identical positive charges are shown in Figure.

(a)

(b)

Some equipotential surfaces for (a) a dipole, (b) two identical positive charges.
18. (a) If the field were not normal to the equipotential surface, it would have nonzero component along the surface. To move a unit test charge against the direction of the component of the field, work would have to be done. But this is in contradiction to the definition of an equipotential surface: there is no potential difference between any two points on the surface and no work is required to move a test charge on the surface. [2]

[1]
(b) Total electrostatic potential energy of system

$$
\begin{align*}
& U=U_{12}+U_{23}+U_{31} \\
& =\frac{1}{4 \pi \varepsilon_{o}}\left[\frac{q(-4 q)}{a}+\frac{(-4 q)(2 q)}{a}+\frac{q(2 q)}{a}\right] \\
& =-\frac{1}{4 \pi \varepsilon_{o}}\left[\frac{10 q^{2}}{a}\right] \tag{1}
\end{align*}
$$

$\therefore$ Work done to dissociate the system
$W=-U$
$=\frac{1}{4 \pi \varepsilon_{o}}\left[\frac{10 q^{2}}{a}\right]$
19. (a)


Let, $V\left(r_{1}\right)$ and $V\left(r_{2}\right)$ be the electric potentials of the field E at the points having position vectors $\overrightarrow{\mathrm{r}}_{1}$ and $\vec{r}_{2}$.
Work done in bringing $\mathrm{q}_{1}$ from infinity to
$\mathrm{r}_{1}$ against the external field $=\mathrm{q}_{1} \mathrm{~V}\left(\overrightarrow{\mathrm{r}}_{1}\right)$.
Work done in bringing $\mathrm{q}_{2}$ from infinity to
$\mathrm{r}_{2}$ against the external field $=\mathrm{q}_{2} \mathrm{~V}\left(\overrightarrow{\mathrm{r}}_{2}\right)$.
Work done on $\mathrm{q}_{2}$ against the force exerted
by $\mathrm{q}_{1}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}\right)$
Here, $\mathrm{r}_{12}$ is distane between $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$. So, the total potential energy of the system = Work done in assembling the two charges.

$$
\begin{equation*}
\mathrm{U}=\mathrm{q}_{1} \mathrm{~V}\left(\overrightarrow{\mathrm{r}}_{1}\right)+\mathrm{q}_{2} \mathrm{~V}\left(\overrightarrow{\mathrm{r}}_{2}\right)+\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}\right) \tag{2}
\end{equation*}
$$

(b) Equipotential surface for $(-q)$ :

(c) Given :
$q_{1}=1+\mu C$
$q_{2}=-1 \mu \mathrm{C}$
$\mathrm{q}_{3}=+2 \mu \mathrm{C}$

$$
\begin{align*}
& \text { Now, Work done }(\mathrm{W})=\text { Energy of the system } \\
& =\mathrm{U}_{12}+\mathrm{U}_{13}+\mathrm{U}_{23} \\
& =\frac{\mathrm{k}}{\mathrm{r}}\left(\mathrm{q}_{1} \mathrm{q}_{2}+\mathrm{q}_{1} \mathrm{q}_{3}+\mathrm{q}_{2} \mathrm{q}_{3}\right) \\
& =\frac{9 \times 10^{9}}{10 \times 10^{-2}}((+1) \times(-1)+(+1) \times(+2) \\
& =-0.09 \text { Jouls }
\end{align*}
$$

## MULTIPLE CHOICE QUESTIONS

1. A charge $q$ is placed at the centre of the line joining two equal charges $Q$. The system of the three charges will be in equilibrium, if $q$ is equal to
(a) $-\frac{\mathrm{Q}}{2}$
(b) $-\frac{\mathrm{Q}}{4}$
(c) $+\frac{\mathrm{Q}}{4}$
(d) $+\frac{\mathrm{Q}}{2}$
2. Inside a hollow charged spherical conductor, the potential
(a) is constant
(b) varies directly as the distance from the centre
(c) varies inversely as the distance from the centre
(d) varies inversely as the square of the distance from the centre.
3. Two small spheres each carrying a charge $q$ are placed $r$ metre apart. If one of the sphere is taken around the other one in a circular path of radius $r$, the work done will be equal to
(a) Force between them $\times \mathbf{r}$
(b) Force between them $\times 2 \pi r$
(c) Force between them / $2 \pi \mathrm{r}$
(d) zero.
4. Two charged spheres of radii 10 cm and 15 cm are connected by a thin wire. No charge will flow, if they have
(a) The same charged on each
(b) The same potential
(c) The same energy
(d) The same field on their surfaces
5. If a unit positive charge is taken from one point to another over an equipotential surface, then
(a) work is done on the charge
(b) work is done by the charge
(c) work done is constant
(d) No work is done
6. Charges of $+\frac{10}{3} \times 10^{-9} \mathrm{C}$ are placed at each of the four corners of a square of side 8 cm . The potential at the intersection of the diagonals is
(a) $150 \sqrt{2}$ volt
(b) $1500 \sqrt{2}$ volt
(c) $900 \sqrt{2}$ volt
(d) 900 volt
7. In the electric field of a point charge q, a certain charge is carried from point A to $\mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . Then the work done

(a) Is least along the path AB
(b) Is least along the path AD
(c) Is zero along all the paths $\mathrm{AB}, \mathrm{AC}, \mathrm{AD}$ and AE
(d) Is least along AE
8. A conductor with a positive charge
(a) Is always at +ve potential
(b) Is always at zero potential
(c) Is always at negative potential
(d) May be at + ve, zero or-ve potential
9. If E is the electric field intensity of an electrostatic field, then the electrostatic energy density is proportional to
(a) E
(b) $\mathrm{E}^{2}$
(c) $\frac{1}{\mathrm{E}^{2}}$
(d) $\mathrm{E}^{3}$
10. Three particles, each having a charge of 10 $\mu \mathrm{C}$ are placed at the corners of an equilateral triangle of side 10 cm . The electrostatic potential energy of the system is
(Given $\frac{1}{4 \pi} \varepsilon_{0}=9 \times 10^{9} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{c}^{2}$ )
(a) Zero
(b) Infinite
(c) 27 J
(d) 100 J
11. Two equal charges $q$ are placed at a distance of 2 a and a third charge $-2 q$ is placed at the midpoint. The potential energy of the system is
(a) $\frac{q^{2}}{8 \pi \varepsilon_{0} a}$
(b) $\frac{6 q^{2}}{8 \pi \varepsilon_{o} a}$
(c) $-\frac{7 q^{2}}{8 \pi \varepsilon_{0} a}$
(d) $\frac{9 q^{2}}{8 \pi \varepsilon_{0} a}$
12. Potential at a point $x$-distance from the centre inside the conducting sphere of radius $R$ and charged with charge $Q$ is proportional to
(a) $\frac{\mathrm{Q}}{\mathrm{R}}$
(b) $\frac{\mathrm{Q}}{\mathrm{x}}$
(c) $\frac{\mathrm{Q}}{\mathrm{x}^{2}}$
(d) xQ

Answer Keys

1. (b)
2. (a)
3. (d)
4. (b)
5. (d)
6. (b)
7. (c)
8. (d)
9. (b)
10. (c)
11. (c)
12. (a)

## Solutions

1. Suppose in the following figure, equilibrium of charge B is considered. Hence for it's equilibrium $\left|F_{A}\right|=\left|F_{C}\right|$
$\Rightarrow \frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{Q}^{2}}{4 \mathrm{x}^{2}}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \frac{\mathrm{qQ}}{\mathrm{x}^{2}} \Rightarrow \mathrm{q}=\frac{-\mathrm{Q}}{4}$

2. Inside the hollow sphere, at any point the potential is constant.
3. The force is perpendicular to the displacement.
4. Because current flows from higher potential to lower potential.
5. On the equipotential surface, electric field is normal to the charged surface (where potential exists) so that no work will be done.
6. Potential at the centre $O, V=4 \times \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\frac{a}{\sqrt{2}}}$
where $Q=\frac{10}{8} \times 10^{-9} \mathrm{C}$ and $\mathrm{a}=8 \mathrm{~cm}=8 \times 10^{-2} \mathrm{~m}$


Hence,

$$
\begin{equation*}
\mathrm{V}=4 \times 9 \times 10^{9} \times \frac{\frac{10}{3} \times 10^{-9}}{\frac{8 \times 10^{-2}}{\sqrt{2}}}=1500 \sqrt{2} \text { volt } \tag{1}
\end{equation*}
$$

7. $A B C D E$ is an equipotential surface, on equipotential surface no work is done in shifting a charge from one place to another.
8. May be at positive, zero or negative potential, it is according to the way one defines the zero potential.
[1]
9. Electrostatic energy density $\frac{\mathrm{dU}}{\mathrm{dV}}=\frac{1}{2} \mathrm{~K} \varepsilon_{0} \mathrm{E}^{2}$
$\therefore \frac{\mathrm{dU}}{\mathrm{dV}} \propto \mathrm{E}^{2}$
10. For pair of charge $U=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{q_{1} q_{2}}{r}$

$$
\begin{align*}
& \therefore \mathrm{U}_{\text {system }}=\frac{3}{4 \pi \varepsilon_{\mathrm{o}}}\left(\frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{\frac{10}{100}}\right) \\
& =3 \times 9 \times 10^{9} \times \frac{100 \times 10^{-12} \times 100}{10}=27 \mathrm{~J} \tag{1}
\end{align*}
$$

11. $\mathrm{U}_{\text {system }}=\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \frac{(\mathrm{q})(-2 \mathrm{q})}{\mathrm{a}}+\frac{1}{4 \pi \varepsilon_{\mathrm{o}}} \cdot \frac{(-2 \mathrm{q})(\mathrm{q})}{\mathrm{a}}$

$$
+\frac{1}{4 \pi \varepsilon_{0}} \frac{(q)(q)}{2 a}
$$

$$
\begin{equation*}
=-\frac{7 q^{2}}{8 \pi \varepsilon_{0} \mathrm{a}} \tag{1}
\end{equation*}
$$

12. Potential at any point inside the charge spherical conductor equals to the potential at the surface of the conductor i.e. $Q / R$

## Topic 2: Capacitance

## Summary

## Capacitor and Capacitance

- Capacitor: The system of two conductors separated by an insulator is called capacitor.
The device which is used to store charge is known as capacitor. The applied voltage and size of capacitor decides the amount of charge that can be stored i.e., $\mathrm{Q}=\mathrm{CV}$
Two similar connecting plates are placed in capacitor in the front of each other where one plate is connected to the positive terminal and other plate is connected to the negative terminal.
- Capacitance: The ratio of magnitude of charge stored on the plate to potential difference between the plates is called capacitance. It is written as:
$C=\frac{Q}{\Delta V}$
Size, shape, medium and other conductors in surrounding influence the capacitance of a conductor.
Its S.I. unit is farad.
$1 \mathrm{~F}=1 \mathrm{CV}^{-1}$ For a parallel plate capacitor (with vacuum between the plates), $C=\varepsilon_{0} \frac{A}{d}$ where $A$ is the area of each plate and $d$ in the separation between the parallel plates.


Fig. Capacitor

## Effect of Dielectric on Capacitance:

- If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect, called polarization, gives rise to a field in the opposite direction.
- The dielectric is polarised by the field and also the effect is equivalent to two charged sheets with surface charge densities $\sigma_{p}$ and $-\sigma_{p}$.
- The net electric field inside the dielectric and hence the potential difference between the plates
is thus reduced. Consequently, the capacitance C increases from its value $\mathrm{C}_{\mathrm{o}}$ when there is no medium (vacuum),
$\mathrm{C}=\mathrm{KC}_{\mathrm{o}}$ where $\mathrm{K}=\frac{\varepsilon}{\varepsilon_{0}}$ is the dielectric constant of the insulating substance.


## Types of capacitor:

- Parallel plate capacitor: $\mathrm{C}=\mathrm{K} \varepsilon_{0} \frac{\mathrm{~A}}{\mathrm{~d}}$
- Cylindrical capacitor: $\mathrm{C}=2 \pi \mathrm{~K} \varepsilon_{0} \frac{1}{\ln (\mathrm{~b} / \mathrm{a})}$
- Spherical capacitor: $\mathrm{C}=4 \pi \mathrm{~K} \varepsilon_{0}\left(\frac{\mathrm{ab}}{\mathrm{b}-\mathrm{a}}\right)$


## Combination of Capacitors

- For capacitors in the series combination, the total capacitanceCisgivenby $\frac{1}{\mathrm{C}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}+\frac{1}{\mathrm{C}_{3}}+\ldots . \frac{1}{\mathrm{C}_{\mathrm{n}}}$
- In the parallel combination, the total capacitance C is $\mathrm{C}=\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3} \ldots \ldots \mathrm{C}_{\mathrm{n}}$, where $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3} \ldots \ldots$ are individual capacitances.
- Capacitors connected in series have the same charges and when connected in parallel have the same voltage.
- Potential across capacitor remains same if the battery is connected but if it is disconnected then chargeremainsthesamewhichisstoredincapacitor.


## Electrical Energy Stored in a Capacitor:

- The energy U stored in a capacitor of capacitance $C$, with charge $Q$ and voltage $V$ is $\mathrm{U}=\frac{1}{2} \mathrm{QV}=\frac{1}{2} \mathrm{CV}^{2}=\frac{1}{2} \frac{\mathrm{Q}^{2}}{\mathrm{C}}$.
- The electric energy density (energy per unit volume) in a region with electric field is $\frac{1}{2} \varepsilon_{0} \mathrm{E}^{2}$.
- Electric density is alternatively known as electrostatic pressure.


## Van De Graaff Generator:

- A Van de Graaff generator consists of a large spherical conducting shell (a few meters in diameter).
- There are two pulleys, one at ground level and one at the center of the shell. Both of them are wounded around by a long and narrow endless belt of insulating material.
- The motor drives the lower pulley which keeps moving this belt continuously.
- At ground level to the top, it continuously carries the positive charge and sprayed on to it by a brush. Then the positive charge is transferred by it to another conducting brush connected to the large shell.
- After the transferring of the positive charge is done, it spreads out uniformly on the outer surface. It can build the voltage difference of as much as 6 to 8 million volts.


Fig. Yande Graff Generator

## PREVIOUS YEARS' examination questions TOPIC 2

## ■ 1 Mark Questions

1. Why should electrostatic field be zero inside a conductor?
[All INDIA 2012]
2. A capacitor has been charged by a dc source. What are the magnitudes of conduction and displacement current, when it is fully charged?
[All INDIA 2013]
3. Define dielectric constant of a medium. What is its S.I. unit?
[DELHI 2015]
4. Predict the polarity of the capacitor in the situation described below:

[All INDIA 2017]
5. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from $2 \mu \mathrm{~F}$ to $\mathrm{X} \mu \mathrm{F}$, the decrease in energy of the capacitor is $2 \times 10^{-2} \mathrm{~J}$. The value of X is -
(a) $1 \mu \mathrm{~F}$
(b) $2 \mu \mathrm{~F}$
(c) $3 \mu \mathrm{~F}$
(d) $4 \mu \mathrm{~F}$
[DELHI TERM I, 2022]
6. A car battery is charged by a 12 V supply, and energy stored in it is $7.20 \times 10^{5} \mathrm{~J}$. The charge passed through the battery is -
(a) $6.0 \times 10^{4} \mathrm{C}$
(b) $5.8 \times 10^{3} \mathrm{~J}$
(c) $8.64 \times 10^{6} \mathrm{~J}$
(d) $1.6 \times 10^{5} \mathrm{C}$
[DELHI TERM I, 2022]
7. The capacitors, each of $4 \mu \mathrm{~F}$ are to be connected in such a way that the effective capacitance of the combination is $6 \mu \mathrm{~F}$. This can be achieved by connecting.
(a) All three in parallel
(b) All three in series
(c) Two of them connected in series and the combination in parallel to the third
(d) Two of them connected in parallel and the combination in series to the third
[DELHI 2023]

## ■ 2 Marks Questions

8. Figure shows two identical capacitors, $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ each of $1 \mu F$ capacitance connected to a battery of 6 V . Initially switch ' S ' is closed. After sometime ' $S$ ' is left open and dielectric slabs of dielectric constant $K=3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?

[DELHI 2011]
9. A slab of material of dielectric constant $K$ has the same area as that of the plates of a parallel plate capacitor but has the thickness $2 \frac{d}{3}$, where $d$ is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.
[DELHI 2011]
10. A capacitor of unknown capacitance is connected across a battery of $V$ volts. The charge stored in it is $360 \mu \mathrm{C}$. When potential across the capacitor is reduced by 120 V , the charge stored in it becomes $120 \mu \mathrm{C}$.
Calculate:
(i) The potential $V$ and the unknown capacitance C .
(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V ?
[DELHI 2011]
11. A parallel plate capacitor of capacitance $C$ is charged to a potential $V$. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor
[All INDIA 2014]
12. A capacitor ' $C$ ' a variable resistor ' $R$ ' and a bulb ' $B$ ' are connected in series to the ac mains in circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance $R$ to be the same; (ii) the resistance $R$ is increased keeping the same capacitance?

[DELHI 2014]
13. Two capacitors of unknown capacitance $C$ 1and $C 2$ are connected first in series and then in parallel across a battery of 100 V . If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of $C 1$ and $C 2$. Also calculate the charge on each capacitor in parallel combination.
[DELHI 2015]
14. The space between the plates of a parallel plate capacitor is completely filled in two ways. In the first case, it is filled with a slab of dielectric constant K. In the second case, it is filled with two slabs of equal dimensions but dielectric constants $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$, respectively as shown in the figure. The capacitance of the capacitor is same in the two cases. Obtain the relationship between $\mathrm{K}, \mathrm{K}_{1}$ and $\mathrm{K}_{2}$.

[DELHI 2020]

## ■ 3 Marks Questions

15. Three circuits, each consisting of a switch ' $S$ ' and two capacitors, are initially charged, as shown in the figure. After the switch has been closed, in which circuit will the charge on the left-hand capacitor (i) increase, (ii) decrease and (iii) remains same? Give reasons.

[All India 2015]
16. Two parallel plate capacitors $X$ and $Y$ have the same area of plates and same separation between them. $X$ has air between the plates while $Y$ contains a dielectric medium of $k=4$.

(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is $4 \mu F$.
(ii) Calculate the potential difference between the plates of $X$ and $Y$.
(iii) Estimate the ratio of electrostatic energy stored in $X$ and $Y$.
[DELHI 2015]
17. The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9 . Bywhat factor does the applied potential difference change?
[All INDIA 2017]
18. Two identical parallel plate capacitors $A$ and $B$ are connected to a battery of $V$ volts with the switch $S$ closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant $K$. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.

[All INDIA 2017]
19. A thin conducting spherical shell of radius $R$ has charge $Q$ spread uniformly over its surface. Using Gauss's law; derive an expression for an electric field at a point outside the shell. Draw a graph of electric field $\mathrm{E}(\mathrm{r})$ with distance r from the centre of the shell for $0 \leq r \leq \infty$
[DELHI 2017]
20. Three identical capacitors $C_{1}, C_{2}$ and $C_{3}$ of capacitance $6 \mathrm{p} F$ each are connected to a 12 V battery as shown.


Find (i) charge on each capacitor (ii) equivalent capacitance of the network (iii) energy stored in the network of capacitors
[DELHI 2017]

## ■ 4 Marks Question

21. A capacitor is a system of two conductors separated by an insulator. The two conductors have equal and opposite charges with a potential difference between them. The capacitance of a capacitor depends on the geometrical configuration (shape, size and separation) of the system and also on the nature of the insulator separating the two conductors. They are used to store charges. Like resistors, capacitors can be arranged in series or parallel or a combination of both to obtain desired value of capacitance.
(i) Find the equivalent capacitance between points A and B in the given diagram.

(ii) A dielectric slab is inserted between the plates of a parallel plate capacitor. The electric field between the plates decreases. Explain.
(iii) A capacitor A of capacitance C, having charge $Q$ is connected across another uncharged capacitor B of capacitance 2 C . Find an expression for (a) the potential difference across the combination and (b) the charge lost by capacitor A.

OR
(iii) Two slabs of dielectric constants 2 K and K fill the space between the plates of a parallel plate capacitor of plate area A and plate separation d as shown in figure. Find an expression for capacitance of the system.

[DELHI 2023]

## ■ 5 Marks Questions

22. Draw a labeled diagram of Van de Graff generator. State its working principle to show how by introducing a small charged sphere into a larger sphere, a large amount of charge can be transferred to the outer sphere. State the use of this machine and also point out its limitations.

Or
(a) Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$
(b) Consider two hollow concentric spheres, $S_{1}$ and $S_{2}$, enclosing charges $2 Q$ and $4 Q$ respectively as shown in the figure
(i) Find out the ratio of the electric flux through them.
(ii) How will the electric flux through the sphere $S_{1}$ change if a medium of dielectric constant ' $\varepsilon$ ' is introduced in the space inside $S_{1}$ in place of air? Deduce the necessary expression.

[DELHI 2014]
23. (a) Distinguish, with the help of a suitable diagram, the difference in the behavior of a conductor and a dielectric placed in an external electric field. How does polarized dielectric modify the original external field?
(b) A capacitor of capacitance $C$ is charged fully by connecting it to a battery of emf $E$. It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change?
(i) Charge stored by the capacitor.
(ii) Field strength between the plates.
(iii) Energy stored by the capacitor.

Justify your answer in each case.
[All INDIA 2016]
24. A parallel-plate capacitor is charged to a potential difference $V$ by a dc source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, a state with reason how the following change:
(i) The electric field between the plates,
(ii) Capacitance and
(iii) Energy stored in the capacitor
[DELHI 2017]
25. Explain the principle of a device that can build up high voltage of the order of a few million volts.
Draw a schematic diagram and explain the working of this device.
Is there any restriction on the upper limit of the high voltage set up in this machine? Explain.
[DELHI 2012]
26. (a) Describe briefly the process of transferring the charge between the two plates of a parallel plate capacitor when connected to a battery. Derive an expression for the energy stored in a capacitor.
(b) Aparallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor.
[DELHI 2019]

## Solutions

1. In a conductor charges reside on its surface, there are no charges present inside a conductor. Hence electric field inside is zero.
2. When capacitor is fully charged it maintains a constant voltage and charge will also be constant. Since current is defined as rate of change of charge it will be zero.

Conduction current, $I_{C}=C \frac{d V}{d t}$
Since, $V$ is constant, $\frac{d V}{d t}=0$
$\Rightarrow \mathrm{I}_{\mathrm{C}}=\mathrm{C} \cdot \frac{\mathrm{dV}}{\mathrm{dt}}=0$
Displacement current, $I_{D}=\varepsilon_{0} \frac{d\left(\frac{q}{\varepsilon_{0}}\right)}{d t}$
Since $q$ is constant, $d\left(\frac{q}{\varepsilon_{0}}\right)=0$
$\Rightarrow I_{D}=\varepsilon_{0} \frac{d\left(\frac{q}{\varepsilon_{0}}\right)}{d t}=0$
3. Dielectric constant (or relative permittivity) of a dielectric is the ratio of the absolute permittivity ( $\varepsilon$ ) of a medium to the absolute permittivity of free $\operatorname{space}\left(\varepsilon_{0}\right) \mathrm{K}=\frac{\varepsilon}{\varepsilon_{0}}$, It is unit less quantity.
4. According to Lenz law the polarity of induced emf is such that it opposes the cause of its production so the polarity of the capacitor is as shown


Fig.: Lenz Law
[1/2]
5. (d) Change in capacitance is related with voltage and energy through the formula-
$\Delta \mathrm{U}=\frac{1}{2} \mathrm{~V}^{2}\left(\mathrm{C}_{2}-\mathrm{C}_{1}\right)$
Taking $\mathrm{C}_{2}$ or new capacitance as X . Keeping the values in the formula to find the value of X .
Multiplying 2 on Left Hand Side
$-2 \times 10^{-2} \times 2=2002 \times(\mathrm{x}-2)$
$\Rightarrow-4 \times \frac{10^{-2}}{4} \times 10^{4}=\mathrm{x}-2 \times 10^{-6}$
$\Rightarrow \mathrm{x}=2 \times 10^{-6}-1 \times 10^{-6} \Rightarrow 1 \times 10^{-6}$ Farad
6. (c) We know that Energy, $U=$ Charge (Q) $X$ potential Difference (DV)
Given, Energy, U = $7.20 \times 10^{5} \mathrm{~J}$
Also, Potential Difference (DV) $=12 \mathrm{~V}$
Hence, $Q=\frac{7.20 \times 10^{5}}{12}=0.6 \times 10^{5}=6 \times 10^{4} \mathrm{C}$
7. (c) Each capacitor is of $4 \mu \mathrm{~F}$.

To get effective capacitance of $6 \mu \mathrm{~F}$, we should connect two of them in series and the combination in parallel to the third capacitor

$\mathrm{C}_{\mathrm{AB}}=(4+2) \mu \mathrm{F}=6 \mu \mathrm{~F}$.
8. In $C_{2}$ : Charge $Q_{o}=C_{\mathrm{D}} \mathrm{V}_{\mathrm{D}}$

Where, $C_{\mathrm{D}}=K C=$ increases $K$ times
$\mathrm{V}_{\mathrm{D}}=\frac{\mathrm{V}}{\mathrm{K}}=$ decreases $K$ times
In $\mathrm{C}_{1}$ : Charge $Q_{\mathrm{o}}=C_{\mathrm{D}} \mathrm{V}$
Potential V remains the same as 6 V
Charge $Q_{\mathrm{D}}=K C V=K Q$, increases K times

$C=\frac{\varepsilon_{0} A}{\frac{2 d / 3}{K}+\frac{d / 3}{1}}=\frac{3 \varepsilon_{0} A K}{d(2+K)}$
10. (i) Initial voltage, $V_{1}=V$ volts and charge stored, $Q_{1}=360 \mu C$
$Q_{1}=\mathrm{C} V_{1}$
Charged potential, $V_{2}=V-120$
$Q_{2}=120 \mu C$
$Q_{2}=\mathrm{C} V_{2}$
By dividing (2) from (1), we get
$\frac{Q_{1}}{Q_{2}}=\frac{C V_{1}}{C V_{2}}$
$\frac{360}{120}=\frac{V}{V-120}$
$V=180$ Volts

$$
C=\frac{q_{1}}{V_{1}}=\frac{360 \times 10^{-6}}{180}
$$

$$
\begin{equation*}
=2 \times 10^{-6} F=2 \mu F \tag{1}
\end{equation*}
$$

(ii) If the voltage applied had increased by 120 $V$, then
$V a=180+120=300 \mathrm{~V}$
Hence, charge stored in the capacitor,

$$
\begin{equation*}
Q_{a}=\underline{C V_{a}}=2 \times 10^{-6} \times 300=600 \mu C \tag{1}
\end{equation*}
$$

11. $U_{\text {initial }}=\frac{1}{2} C V^{2}+0=\frac{1}{2} C V^{2}$

After connecting common potential

$$
\begin{align*}
& V_{\text {common }}=\frac{\left(C_{1} V_{1}+C_{2} V_{2}\right)}{C_{1} C_{2}}=\frac{V}{2} \\
& U_{\text {Final }}=\frac{1}{2} C\left(\frac{V}{2}\right)^{2}+\frac{1}{2} C\left(\frac{V}{2}\right)^{2}=\frac{1}{4} C V^{2}  \tag{1}\\
& \therefore \frac{U_{\text {Initial }}}{U_{\text {Final }}}=\frac{\left(\frac{1}{4} C V^{2}\right)}{\left(\frac{1}{2} C V^{2}\right)} \\
& \frac{U_{\text {Initial }}}{U_{\text {Final }}}=1: 2 \tag{1}
\end{align*}
$$

12. (i) As the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease $(V=Q / C)$. As a result, the potential drop across the bulb will increase (since both are connected in series'). So, its brightness will increase.
(ii) As the resistance ( $R$ ) is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease (since both are connected in series). So, its brightness will decrease.
[1]
13. When the capacitors are connected in parallel,

Equivalent capacitance, $C_{\mathrm{p}}=C_{1}+C_{2}$
The energy stored in the combination of the capacitors, $E_{p}=\frac{1}{2} C_{p} V^{2}$
$E_{p}=\frac{1}{2}\left(C_{1}+C_{2}\right)(100)^{2}=0.25 J$
$\Rightarrow\left(C_{1}+C_{2}\right)=5 \times 10^{-5}$
When the capacitors are connected in series.
Equivalent Capacitance, $C_{s}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}$
The energy stored in the combination of the capacitors, $E_{s}=\frac{1}{2} C_{s} V^{2}$
$\Rightarrow E_{s}=\frac{1}{2} \frac{C_{1} C_{2}}{C_{1}+C_{2}}(100)^{2}=0.045 J$
$\Rightarrow \frac{1}{2} \frac{C_{1} C_{2}}{5 \times 10^{-5}}(100)^{2}=0.045 \mathrm{~J}$
$\Rightarrow C_{1} C_{2}=0.045 \times 10^{-4} \times 5 \times 10^{-5} \times 2=4.5 \times 10^{-10}$
$\Rightarrow\left(C_{1}-C_{2}\right)^{2}=\left(C_{1}+C_{2}\right)^{2}-4 C_{1} C_{2}$
$\Rightarrow\left(C_{1}-C_{2}\right)=\sqrt{7 \times 10^{-10}}=2.64 \times 10^{-5}$
$C_{1}=C_{2}=2.64 \times 10^{-5} \quad(2)$
Solving (1) and (2), we get
$C_{1}=38.2 \mu F$ And $C_{2}=0.12 \mu F$
When the capacitors are connected in parallel, the charge on each of them can be obtained as follows:
$Q_{1}=C_{1} V=382.2 \times 10^{-6} \times 100=38.210^{-4} \mathrm{C}$
$Q_{2}=C_{2} V=0.12 \times 10^{-6} \times 100=0.1210^{-4} C$
14. Here, $\mathrm{C}_{1}=\frac{\mathrm{k} \varepsilon_{0} \mathrm{~A}}{\mathrm{~d}}$
$\mathrm{C}_{2}=$ Parallel combination of two capacitors,

$$
\begin{equation*}
=\frac{\mathrm{k}_{1} \varepsilon_{\mathrm{o}}(\mathrm{~A} / 2)}{\mathrm{d}}+\frac{\mathrm{k}_{2} \varepsilon_{\mathrm{o}}(\mathrm{~A} / 2)}{\mathrm{d}}=\frac{\varepsilon_{0} \mathrm{~A}}{2 \mathrm{~d}}\left(\mathrm{k}_{1}+\mathrm{k}_{2}\right) \tag{1}
\end{equation*}
$$

As per question, capacitance of the capacitor is same in two cases, $\mathrm{So}, \mathrm{C}_{1}=\mathrm{C}_{2}$
Hence, $k=\frac{\mathrm{k}_{1}+\mathrm{k}_{2}}{2}$
15. When charged capacitors are connected to each other then the charge will flow from the capacitor with higher potential towards the capacitor with lower potential untill a common potential is reached.
(a) In fig. (a) the potential of both the capacitor is same so the charge on left hand capacitor remains the same
(b) In fig. (b) the potential of left hand capacitor is high so charge from $6 Q$ to $3 Q$. Therefore charge on left hand capacitor will decrease.
[1]
(c) In fig. (c) the potential of left hand capacitor is low so charge will flow from $3 Q$ to $6 Q$. Therefore charge on left hand capacitor will increase.
16. (i) Let the capacitance of $X$ be $C_{1}$ and capacitance of $Y$ be $C_{2}$
$C_{1}=\frac{\varepsilon_{0} A}{d}$
$C_{2}=\frac{\varepsilon_{r} \varepsilon_{0} A}{d}$
$\frac{C_{1}}{C_{2}}=\frac{1}{\varepsilon_{r}}$
$\Rightarrow C_{2}=\varepsilon_{r} C_{1}$
$C_{1}=C$
$C_{2}=4 C \quad\left(\because \varepsilon_{r}=4\right)$
Since two capacitance are connected in series so, equivalent capacitance will be
$\frac{1}{C_{e q}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}$
$C_{e q}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}$
$4 \mu F=\frac{C \times 4 C}{C+4 C}$
$\Rightarrow C=5 \mu F$
So, $C_{1}=5 \mu F$ and $C_{2}=20 \mu F$
[1/2]
(ii) $C_{\text {eq }} V_{\text {net }}=Q_{\text {Total }}$
$Q_{\text {Total }}=60 \mu C$
Since in series configuration charge on each capacitor is equal.
Hence, $Q_{1}=Q_{2}=Q_{\text {Total }}=60 C$
[1/2]
Using $Q=C V$
$V_{1}=\frac{Q_{1}}{C_{1}}=\frac{60 \mu C}{5 \mu F}=12 \mathrm{~V}$

$$
\begin{align*}
& V_{2}=\frac{Q_{2}}{C_{2}}=\frac{60 \mu C}{20 \mu F}=3 \mathrm{~V}  \tag{1/2}\\
& \text { (iii) } U_{1}=\frac{1}{2} \frac{Q_{1}^{2}}{C_{1}}=\frac{1}{2} \frac{(60 \mu C)^{2}}{5 \mu F}=360 \mu \mathrm{~J} \\
& U_{2}=\frac{1}{2} \frac{Q_{2}^{2}}{C_{2}}=\frac{1}{2} \frac{(60 \mu \mathrm{C})^{2}}{20 \mu F}=900 \mu \mathrm{~J}  \tag{1/2}\\
& \Rightarrow \frac{U_{1}}{U_{2}}=\frac{4}{1} \\
& V_{1}: V_{2}:: 4: 1 \tag{1/2}
\end{align*}
$$

17. Let the heat dissipated per unit time.
$\mathrm{H}=\frac{(\mathrm{V})^{2}}{\mathrm{R}}$
$\mathrm{H}=\frac{(12)^{2}}{6}=24 \mathrm{~J} / \mathrm{sec}$
The new heat dissipated per unit time ( $H=H X$ $9=216 \mathrm{~J} / \mathrm{Sec}$
Let the new voltage be $V$
$\frac{(\mathrm{V})^{2}}{\mathrm{R}}=216$
$(V)^{2}=216 \times 6$
$V=36$ volt
18. Net capacitance before filling the gap with dielectric slab is given by
$C_{\text {Initial }}=A+B$
Net capacitance at here filling the gap with dielectric slab of dielectric constant
$C_{\text {fnal }}=K(A+B) \quad$... (ii)
Energy stored by capacitor is given by $U=\frac{Q^{2}}{2 C}$
[1/2]
So energy stored in capacitor Combination before introduction of dielectric slab
$U_{\text {initial }}=\frac{Q^{2}}{(A+B)}$
Energy stored in combination after introduction of dielectric slab
$U_{\text {final }}=\frac{Q^{2}}{K(A+B)}$
Ratio of energy stored $\frac{U_{\text {initial }}}{U_{\text {final }}}=\frac{K}{1}$
[1⁄2]
19. According to Gauss law:
$\varepsilon_{o} E \oint d A=q$
Where, $q$ is the point charge $E$ is electric field due to the point charge $d A$ is a small area on the Gaussian surface at any distance and $\varepsilon_{0}$ is the proportionality constant.
For a spherical shell at distance $r$ from the point charge, the integral $\oint d A$ is merely the sum of all differential of dA on the sphere.
Therefore, $\oint d A=4 \pi r^{2}$
$\varepsilon_{o} E\left(4 \pi r^{2}\right)=q$
or,
$E=\frac{q}{\varepsilon_{o}\left(4 \pi r^{2}\right)}$
Therefore, for a thin conducting spherical shell of radius $R$ and charge $Q$. spread uniformly over its surface, the electric field at any point outside the shell is

$$
E=\frac{Q}{\varepsilon_{o}\left(4 \pi r^{2}\right)}
$$

Where $r$ is the distance of the point from the centre of the shell

$$
E=\frac{q}{\varepsilon_{o}\left(4 \pi r^{2}\right)}
$$

The graph of electric field $E(r)$ with distance $r$ from the centre of the shell for $0 \leq r \leq \infty \quad[1 / 2]$

[1/2]
Fig. Valuation of Electric field with respect to distance
20. The $12 V$ battery is in parallel with $C_{1}, C_{2}$ and $C_{3} . C_{1}, C_{2}$ are in series with each other while $C_{3}$ is in parallel with the combination formed by $C_{1}$ and $C_{2}$. Total voltage drop across
$C_{3}=12 \mathrm{~V}$
$q_{3}=C V$
Where, $q=$ Charge on the capacitor $C_{1}, C_{2}, C_{3}=$ $6 \mu F$ (Given in the question)
$q_{3}=6 \times 12=72 \mu C$
Voltage drop across $C_{1}$ and $C_{2}$ combined will be 12 V .

Let the voltage drop at $C_{1}=V_{1}$
Let the voltage drop at $C_{2}=V_{2}$
Then, $V=V_{1}+V_{2}$
$V_{2}=\frac{q_{2}}{C}$
$V_{1}=\frac{q_{1}}{C}$
$\frac{q_{1}}{6}+\frac{q_{2}}{6}=12$
As both the capacitors are in series.
$q_{1}=q_{2}=q$
Then,
$q\left\{\frac{1}{6}+\frac{1}{6}\right\}=12$
$q \times \frac{1}{3}=12$
$q=36$ micro coulombs Thus. charge on each of is 36 coulombs.
[1]
21. (i)


In the above circuit diagram, we can remove capacitor with plate number 33' by applying Wheatstone bridge principle.


$$
\begin{equation*}
\mathrm{C}_{\mathrm{eq}}=\frac{\mathrm{C}}{2}+\frac{\mathrm{C}}{2}+\mathrm{C} \tag{1}
\end{equation*}
$$

(ii)


When a dielectric slab is inserted between the charged plates of a capacitor, due to polarization of molecules of dielectric slab an internal electric field is generated in slab in the direction opposite to that of between the plates of capacitor. Hence, net electric field between the plates of capacitor decreases.
$\mathrm{E}_{\text {net }}=\mathrm{E}-\mathrm{E}_{\text {in }}$
(iii) (a) Given $\mathrm{C}_{\mathrm{A}}=\mathrm{C}$

Charge on $\mathrm{C}_{\mathrm{A}}=\mathrm{Q}$
Q = CV
When two capacitors are connected in parallel then
$\mathrm{C}_{\text {eq }}=\mathrm{C}+2 \mathrm{C}=3 \mathrm{C}$
$\mathrm{Q}_{\text {net }}=\mathrm{Q}$ (Because charge will be conserved)
$V^{\prime}=\frac{Q}{3 C}$
Potential difference across each
capacitor $=\frac{\mathrm{Q}}{3 \mathrm{C}}$
(b) Now final charge on capacitor

$$
\mathrm{A}=\mathrm{C} \times \frac{\mathrm{Q}}{3 \mathrm{C}}=\frac{\mathrm{QC}}{3 \mathrm{C}}=\frac{\mathrm{Q}}{3}
$$

