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2011-2024

Mathematics

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PREFACE

Mathematics is a tricky subject. Your basic concepts of Mathematics need to be in place if you want to excel in the Board Examination. At Career Launcher, our goal is not only to maximize your scores in Class XII Mathematics Board Exam, but also to lay a strong foundation in the subject to help you get ahead in your college and professional career. Over the last decade, we all have seen how the question paper pattern of Class XII Mathematics paper has kept changing. Bearing in mind this unpredictable nature of Class XII board papers, we've come up with Chapter-wise and Topic-wise Solved Papers for Mathematics for Class XII - to help you prepare better and face the Boards with confidence.

Exclusively designed for the students of CBSE Class XII by highly experienced teachers, the book provides answers to all actual questions of Mathematics Board Exams conducted from 2011 to 2024. The solutions have been prepared exactly in coherence with the latest marking pattern; after a careful evaluation of previous year trends of the questions asked in Class XII Boards and actual solutions provided by CBSE.

The book follows a three-pronged approach to make your study more focused. The questions are arranged Chapter-wise so that you can begin your preparation with the areas that demand more attention. These are further segmented topic-wise and eventually the break-down is as per the marking scheme. This division will equip you with the ability to gauge which questions require more emphasis and answer accordingly. Apart from this, several value-based questions have also been included.

We hope the book provides the right exposure to Class XII students so that you not only ace your Boards but mold a better future for yourself. And as always, Career Launcher's school team is behind you with its experienced gurus to help your career take wings.

Let's face the Boards with more confidence!

Wishing you all the best,

Team CL

Blueprint & Marks Distribution

Class 12th Mathematics 2024-25 Analysis Unit Wise

Units	Name of Units	No. of Periods	Marks Distribution
Unit-1	Relations and Functions	30	08
Unit-2	Algebra	50	10
Unit-3	Calculus	80	35
Unit-4	Vectors and Three-Dimensional Geometry	30	14
Unit-5	Linear Programming	20	05
Unit-6	Probability	30	08
	Total	**240**	**80**
	Internal Assessment		20

UNIT I: RELATIONS AND FUNCTIONS 30 Periods

Chapter 1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

Chapter 2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

UNIT II: ALGEBRA 50 Periods

Chapter 3. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew-symmetric matrices. Operation on matrices: Addition and multiplication and Multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Noncommutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Chapter 4. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

UNIT III: CALCULUS 80 Periods

Chapter 5. Continuity and Differentiability

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

Chapter 6. Applications of Derivatives

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

Chapter 7. Integrals

Integration as inverse process of differentiation.Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int\frac{dx}{x^2\pm a^2},\int\frac{dx}{\sqrt{x^2\pm a^2}},\int\frac{dx}{\sqrt{a^2-x^2}},\int\frac{dx}{ax^2+bx+c},\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int\frac{px+q}{ax^2+bx+c}dx,\int\frac{px+q}{\sqrt{ax^2+bx+c}},\int\sqrt{a^2\pm x^2dx},\int\sqrt{x^2-a^2}dx,$$

$$\int\sqrt{ax^2+bx+c}\ dx,\int(px+q)\sqrt{ax^2+bx+c}\ dx.$$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

Chapter 8. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, circles/parabolas/ellipses (in standard form only).

Chapter 9. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}+py = q$, where *p* and *q* are functions of *x* or constants.

$\frac{dx}{dy}+px = q$, where *p* and *q* are functions of *y* or constants.

UNIT IV: VECTOR ALGEBRA AND THREE-DIMENSIONAL GEOMETRY 30 Periods

Chapter 10. Vector Algebra

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.

Chapter 11. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines.Cartesian and vector equation of a plane. Angle between two lines

UNIT V: LINEAR PROGRAMMING 20 Periods

Chapter 12. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded and unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit VI: PROBABILITY 30 Periods

Chapter 13. Probability

Conditional probability, multiplication theorem on probability. independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of random variable.

CHAPTER 1

Relations and Functions

[TOPIC 1] Concept of Relations and Functions

Summary

Relation

Definition: If $(a, b) \in R$, we say that a is related to b under the relation R and we write it as aRb.

Domain of a relation: The set of first components of all the ordered pairs which belong to R is the domain of R.

Domain $(R) = \{a \in A : (a,b) \in R \,\forall\, b \in B\}$

Range of a relation: The set of second components of all the ordered pairs which belong to R is the domain of R.

Range of $R = \{b \in B : (a,b) \in R \,\forall\, a \in A\}$

Types of relations:

- **Empty relation:** Empty relation is the relation R from X to Y if no element of X is related to any element of Y, it is given by $R = \varphi \subset X \times Y$.

  For example, let $X = \{2,4,6\}, Y = \{8,10,12\}$

  $R = \{(a,b) : a \in X, b \in Y \text{ and } a + b \text{ is odd}\}$

  R is an empty relation.
- **Universal relation:**Universal relation is a relation R from X to Y if each element of X is related to every element of Y it is given by $R = X \times Y$.

  For example, let $X = \{x, y\}$, $Y = \{x, z\}$

  $R = \{(x, x), (y, z), (y, x), (y, z)\}$

  $R = X \times Y$, so relation R is a universal relation.
- **Reflexive relation:** Reflexive relation R in X is a relation with $(a, a) \in R \;\forall a \in X$.

  For example, let $X = \{x, y, z\}$ and relation R is given as $R = \{(x, x), (y, y), (z, z)\}$

  Here, R is a reflexive relation on X.
- **Symmetric relation:** Symmetric relation R in X is a relation satisfying $(a, b) \in R$ implies $(b, a) \in R$.

  For example, let $X = \{x, y\ z\}$ and relation R is given as

  $$R = \{(x, y), (y, x)\}$$

  Here, R is a symmetric relation on X.
- **Transitive relation:** Transitive relation R in X is a relation satisfying $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, b) \in R$.

  For example, let $X = \{x, y, z\}$ and relation R is given as

  $$R = \{(x, z), (z, y), (x, y)\}$$

  Here, R is a transitive relation on X.
- **Equivalence relation:** It is a relation R in X which is reflexive, symmetric and transitive.

  For example, let $X = \{x, y, z\}$ and relation R is given as $R = \{(x, y), (x, x), (y, x), (y, y), (z, z), (x, z), (z, x), (y, z)\}$

  Here, R is reflexive, symmetric and transitive. So R is an equivalence relation on X.
- Equivalence class $[a]$ containing $a \in X$ for an equivalence relation R in X is the subset of X containing all elements b related to a.

Function

Definition: A rule f which associates each element of a non-empty set A with a unique element of another non-empty set B is called a function.

Types of functions:

- **Injective function:** A function $f : X \to Y$ is one-one (or injective) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \forall x_1, x_2 \in X$.
- **Surjective function:** A function $f : X \to Y$ is onto (or surjective) if given any $y \in Y$, $\exists x \in X$ such that $f(x) = y$.
- **Bijective function:** A function $f : X \to Y$ is one-one and onto (or bijective), if f is both one-one and onto.

- **Composite function:** The composition of functions $f : A \to B$ and $g : B \to C$ is the function $gof : A \to C$ given by $gof(x) = g(f(x)) \forall x \in A$
- **Invertible function:** A function f : $X \to Y$ is invertible if $\exists g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$.
  A function f : $X \to Y$ is invertible if and only if f is one-one and onto.
- **Steps to find inverse of a function**
  Let $f(x) = y$ where $x \in X$ and $y \in Y$
  Solve $f(x) = y$ for x in terms of y.
  Now replace x with $f^{-1}(y)$ in the expression obtain from the above step.
  Finally to find the inverse function of $f f^{-1}(x)$ replace y with x in the expression obtained from the above step.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. State the reason for the relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.
   [DELHI 2011]
2. If the function f : R → R be defined by f(x) = 2x – 3 and g : R → R by g(x) = x^3 + 5, then find the value of $(fog)^{-1}$ (x).
   [ALL INDIA 2015]
3. If R = $\{(x, y) : x + 2y = 8\}$ is a relation on N, Write the range of R.
   [ALL INDIA 2014]
4. R → R is defined by $f(x) = 3x + 2$, define f[f(x)].
   [ALL INDIA 2011]
5. If f: R → R be given by f $(x) = (3 - x^3)^{1/3}$, then fof (x) = ___________ [DELHI 2020]
6. Let set X = {1, 2, 3} and a relation R is defined in X as : R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are
   (a) {(1, 1), (2, 3), (1, 2)}
   (b) {(3, 3), (3, 1), (1, 2)}
   (c) {(1, 1), {3, 3}, (3, 1), (2, 3)}
   (d) {(1, 1), (3, 3), (3, 1), (1, 2)}
   [DELHI Term I, 2022]
7. If R = {(x, y); x, y ∈ Z, $x^2 + y^2 \le 4$} is a relation in set Z, then domain of R is
   (a) {0, 1, 2} (b) {–2, –1, 0, 1, 2}
   (c) {0, –1, –2} (d) {–1, 0, 1}
   [DELHI Term I, 2022]
8. Let X = {x^2 : x ∈N} and the function f : N → X is defined by f(x) = x^2, x ∈ N. Then this function is
   (a) injective only (b) not bijective
   (c) surjective only (d) bijective
   [DELHI Term I, 2022]
9. A function f : R → R defined by f(x) = 2 + x^2 is
   (a) not one-one
   (b) one-one
   (c) not onto
   (d) neither one-one nor onto
   [DELHI Term I, 2022]
10. Let A = {3, 5}. Then number of reflexive relations on A is
    (a) 2 (b) 4
    (c) 0 (d) 8 [DELHI 2023]

2 Marks Question

11. Check if the relation R on the set A = { 1, 2, 3, 4, 5, 6 } defined as R = {(x, y): y is divisible by x} is (i) symmetric (ii) transitive [DELHI 2020]

4 Marks Questions

12. Show that the function f in
    A = $R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}. [DELHI 2013]
13. Let $A = \{1,2,3,......9\}$ and R be the relation in $A \times A$ defined by $(a,b)R(c,d)$.
    If a + d = b + c for (a, b) (c, d) in $A \times A$, prove that R is an equivalence relation. Also obtain the equivalence class $[(2,5)]$. [DELHI 2014]
14. Let A = R – {3} and B = R – {1}. Consider the function f: A → B Defined by f(x) = $\left(\frac{x-2}{x-3}\right)$ show that f is one-one and also hence find f^{-1}.
    [ALL INDIA 2012]
15. Consider $f : R^+ \to [4, \infty]$ given by $f(x) = x^2 + 4$.. Show that f is invertible with the inverse (f^{-1}) of f given by $f^{-1}(y) = \sqrt{y-4}$, where R^+ is the set of all non-negative real numbers.
    [ALL INDIA 2011, 2013]

16. Let $A = R - \{3\}$ and $B = R - \{1\}$ consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find $f^{-1}(x)$.

[DELHI 2012]

17. If the function $f : R \to R$ be given by $f(x)\ x^2 + 2$ and $g : R \to R$ be given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, find fog and gof and hence find fog (2) and gof (–3). **[ALL INDIA 2014]**

18. Show that the relations R on $\mathbb{R}$ defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric.

OR

Prove that the function $f : N \to N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f : N \to S$, where S is range of f.

[DELHI 2019]

19. Prove that the relation R on Z, defined by $R\{(x, y) : (x - y)$ is divisible by 5$\}$ is an equivalence relation. **[DELHI 2020]**

20. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions:

(I) How many relations are possible from B to G?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G?

(III) Let $R : B \to B$ de defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer. **[DELHI 2023]**

6 Marks Questions

21. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right).$$

Hence find

(i) $f^{-1}(10)$

(ii) y if $f^{-1}(y) = \frac{4}{3}$, Where R_+ is the set of all non-negative real numbers.

[DELHI 2017]

22. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a,b)R(c,d)$ if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

[DELHI 2015]

23. Let $f : N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to N$, where S is the range of, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$

[DELHI 2016]

24. Consider $f : R - \left\{\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is objective. Find the inverse and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$

[ALL INDIA 2017]

25. Let and $A = R \times R$ and * be a binary operation on A defined by $(a,b) * (c,d) = (a+c, b+d)$. Show that * Find the identity element for * on A. Also find the inverse of every element $(a, b) \in A$.

[ALL INDIA 2016]

26. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}, \forall x \in \mathbb{R}$ is neither one-one nor onto. Also $g : R \to R$ if is defined as $g(x) = 2x - 1$, find fog (x).

[DELHI 2018]

27. Let $A = \{x \in \mathbf{Z} : 0 \le x \le 12\}$, Show $\{(a, b) : |a - b|$ is divisible by 4} that is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. **[DELHI 2018]**

Solutions

1. Let R = {1, 2, 3}

As $(1,2) \in R$ and $(2,1) \in R$

But $(1,1) \notin R$ **[1]**

So, R is not transitive.

2. Let $y = (fog)(x)$

Let $y = h(x)$

$= f[g(x)] = f(x^3 + 5)$ **[½]**

$= 2x^3 + 7$

Thus, $x = \sqrt[3]{\dfrac{y-7}{2}} = h^{-1}(y)$

$\Rightarrow (fog)^{-1}_{(x)} = \sqrt[3]{\dfrac{y-7}{2}}$ **[½]**

3. $R = \{(x, y) : x + 2y = 8\}$ is a relation on N

Then we can say $2y = 8 - x$

$y = 4 - \dfrac{x}{2}$

so we can put the value of x, x = 2, 4, 6 only

we get y = 3 at x = 2

we get y = 2 at x = 4

we get y = 1 at x = 6 **[1]**

so range = {1, 2, 3}

4. We have

$f(x) = 3x + 2$

Now, $f[f(x)] = 3(3x + 2) + 2$

$= 9x + 6 + 2 = 9x + 8$ **[1]**

5. As given that,

$f(x) = (3 - x^3)^{1/3}$, If $f : R \to R$,

So, $fof(x) = f\{f(x)\}$

$= \left\{(3 - x^3)^{1/3}\right\}$

$= \left[3 - \left\{(3 - x^3)^{1/3}\right\}^3\right]^{1/3}$

$= \left[3 - (3 - x^3)\right]^{1/3}$

$= [3 - 3 + x^3]^{1/3} = [x^3]^{1/3} = x.$ **[1]**

6. (c) {(1,1), (3,3), (3,1), (2,3)}

(i) R is reflexive if it contains {(1,1), (2,2) & (3,3)

Since (2,2) ∈ R. So, we need to add (1,1) and (2,2) to make R reflexive

(ii) R is symmetric if it contains (2,2), (1,3), (3,1), (3,2) & (2,3)

Since (2,2), (1,3), (3,2) € R. SO we need to add (3,1) & (2,3).

Hence, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are {(1,1), (3,3), (3,1), (2,3)} **[1]**

7. (b) (–2, –1, 0, 1, 2)

We have $R = \{(x, y) : x, y \in Z, x^2 + y^2 \le 4\}$.

Let x = 0

$\therefore \quad x^2 + y^2 \le 4$

$\Rightarrow \quad y^2 \le 4$

$\Rightarrow \quad y = 0, \pm1, \pm2$

Let $\quad x = \pm1$

$\therefore \quad x^2 + y^2 \le 4$

$\Rightarrow \quad y^2 \le 3$

$\Rightarrow \quad y = 0, \pm1$

Let $\quad x = \pm2$

$\therefore \quad x^2 + y^2 \le 4$

$\Rightarrow \quad y^2 \le 0$

$\Rightarrow \quad y = 0$

$\therefore$ R = {(0, 0), (0, –1), (0, 1), (0, –2), (0, 2), (–1, 0), (1, 0), (1, 1), (1, –1), (–1, 1), (2, 0), (–2, 0)}

$\therefore$ Domain of R = {x:(x, y) ∈ R} = {0, –1, 1, –2, 2}. **[1]**

8. (a) Injective

Let $x_1, x_2 \in N$

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

$\Rightarrow x_1 = x_2$

$(x_1 + x_2) \ne 0$ as $x_1, x_2 \in N$

Hence, f(x) is injective.

Also, the elements like 2 and 3 have no pre-image in N. Thus f(x) is not surjective. **[1]**

9. (b) Neither one-one nor onto

Given, $f(x) = 2 + x^2$

For one-one, $f(x_1) = f(x_2)$

$$2 + x_1^2 = 2 + x_2^2$$

$$x_1^2 = x_2^2$$

$x_1 = \pm x_2$

$x_1 = x_2$ and $x_1 = -x_2$

Hence, f(x) is not one-one.

For onto,

Let f(x) = y such that $y \in R$

Therefore, $x^2 = y - 2$

$$\Rightarrow x = \pm\sqrt{y-2}$$

Put y = –3, we get -

$$\Rightarrow x = \pm\sqrt{3-2} = \pm 5$$

Which is not possible as root of negative is not a real number.

Hence, x is not real. So, f(x) is not onto. [1]

10. (b) Given A = {3, 5} ⇒ n(A) =2

We know, Number of reflexive relations

$$= 2^{n^2-n} = 2^{2^2-2} = 4 \quad [1]$$

11. As given that :

A = {1, 2, 3, 4, 5, 6}

R = {(x, y) : y is divisible by x.}

(i) Let $x, y \in A$

If $(x, y) \in R \Rightarrow$ y is divisible by x.

It is not necessary that x is divisible by y.

$\Rightarrow (y, x) \not\in R$, [1]

Ex : $(2, 4) \in R$

$(4, 2) \not\in R$

(ii) Let $x, y, z \in A$

$(x, y) \in R \Rightarrow$ y is divisible by x.

$(y, z) \in R \Rightarrow$ z is divisible by y.

$(x, z) \in R \Rightarrow$ z is divisible by x.

So, R is transitive. [1]

Ex : $(2, 4) \in R$, $(4, 16) \in R$,

$\Rightarrow (2, 16) \in R$

12. Given $f(x) = \dfrac{4x+3}{6x-4}$

For f is one-one

Let $x_1, x_2 \in R$

$$f(x_1) = f(x_2)$$

$$\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4} \quad [1]$$

$$\therefore (4x_1+3)(6x_2-4) = (4x_2+3)(6x_1-4)$$

$$\therefore 24x_1x_2 - 16x_1 + 18x_2 - 12$$

$$\therefore -16x_1 - 18x_1 = -16x_2 - 18x_2$$

$$\therefore -34x_1 = -34x_2 \Rightarrow x_1 = x_2$$

f is one-one

Let y be any element of R.

$$y = f(x) \Rightarrow y = \frac{4x+3}{6x-4} \quad [1]$$

$$\Rightarrow 6xy - 4y = 4x + 3 \Rightarrow -4y - 3 = 4x - 6xy$$

$$\Rightarrow -4y - 3 = (4-6y)x \Rightarrow \frac{-(4y+3)}{-(6y-4)} = x$$

$$\Rightarrow x = \frac{4y+3}{6xy-4}$$

$$f(x) = \frac{4x+3}{6x-4}$$

$$f\left(\frac{4y+3}{6y-4}\right) = \frac{4\left(\frac{4y+3}{6y-4}\right)+3}{6\left(\frac{4y+3}{6y-4}\right)-4} \quad [1]$$

$$= \frac{\frac{16y+12+18y-12}{6y-4}}{\frac{24y+18-24y+16}{6y-4}} = \frac{34y}{34} = y$$

$$F^{-1}(y) = \frac{4y+3}{6y-4};\ F^{-1}(x) = \frac{4x+3}{6x-4} \quad [1]$$

13. **Reflexive:** Let $a, b \in N$

$$\because a + b = b + a \Rightarrow (a,b)R(a,b) \quad [1]$$

$\therefore$ R is reflexive

Symmetric: Let a, b, c, d $\in$ N

$$\because (a,b)R(c,d) \Rightarrow a + d = b + c$$

$\Rightarrow b+c=a+d \quad \Rightarrow c+b=d+a$ [1]

$\Rightarrow (c,d)R(a,b)$

$\therefore$ R is symmetric

Transitive: $a,b,c,d,e,f \in N$

Let $(a,b)R(c,d) and (c,d)R(e,f)$

$\Rightarrow a+d=b+c \; and \; c+f=d+e$

$\Rightarrow a+d+c+f=b+c+d+e$ [1]

$\Rightarrow a+f=b+e \Rightarrow (a,b)R(e,f)$

$\therefore$ R is transitive.

$\because$ R is reflexive, symmetric and transitive.
Hence, R is an equivalence relation.
For equivalence class. [1]

14. Given: For f is one-one

Let $x_1, x_2 \in R$

$f(x_1)=f(x_2)$ [1]

$\therefore \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$

$\therefore (x_1-2)(x_2-3)=(x_1-3)(x_2-2)$

$\therefore x_1x_2-3x_1-2x_2+6=x_1x_2-2x_1-3x_2+6$

$\therefore -3x_1+2x_1=-3x_2+2x_2$

$-x_1=-x_2 \quad \Rightarrow \quad x_1=x_2$

$\therefore f$ is one - one [1]

Now for f is onto
Let y be any element of R

$\therefore y=f(x)$

$\therefore y=\frac{x-2}{x-3}$

$\therefore y=\frac{x-2-1+1}{x-3}$

$\therefore y=\frac{(x-3)}{x-3}+\frac{1}{x-3}$

$\therefore y-1=\frac{1}{x-3}$

$\therefore x-3=\frac{1}{y-1}$

$\therefore x=\frac{1}{y-1}+3$

$\therefore (y-1)(x-3)=1$

$\therefore x=\frac{1+3y-3}{y-1}$

$f(x)=\frac{x-2}{x-3}$

$f\left(\frac{3y-2}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}$ [1]

$=\frac{\frac{3y-2-2y+2}{y-1}}{\frac{3y-2-3y+3}{y-1}}=\frac{y}{1}=y$

Thus, f is onto

$f^{-1}(y)=\frac{3y-2}{y-1};$

$f^{-1}(x)=\frac{3x-2}{x-1}$ [1]

15. $\mathrm{f}(\mathrm{x})=\mathrm{x}^2+4$

Let $\mathrm{f}(\mathrm{x})=\mathrm{f}(\mathrm{y})$

$\mathrm{x}^2+4=\mathrm{y}^2+4$
$\mathrm{x}^2=\mathrm{y}^2$
$\therefore \mathrm{x}=\mathrm{y}$ [1]

As $\mathrm{x}=\mathrm{y}\in\mathrm{R}^+$

Hence, f is a one-one function.

For $\mathrm{y}\in[4,\infty)$,

Let y = x^2+4 [1]

$\therefore \mathrm{x}^2=\mathrm{y}-4\geq 0$

$\therefore \mathrm{x}=\sqrt{\mathrm{y}-4}\geq 0$

Thus, for any $y \in R$.

There exist $x = \sqrt{y-4} \in R$ such that

$$f(x) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y - 4 + 4 = y$$

Hence f is onto.

Therefore, f is one-one and onto. [1]

Now, calculate f^{-1}.

Let, $g(y) = \sqrt{y-4}$

(go f) (x)

$$= g\left[f(x)\right] = g(x^2+4) = \sqrt{(x^2+4)-4} = x$$

(fog) (y)

$$= f\left[g(y)\right] = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4$$

$$= y - 4 + 4 = y$$

$\therefore$ gof = fog [1]

Hence, f is invertible

$$f^{-1}(y) = g(y) = \sqrt{y-4}$$

16. Now, $f(x_1) = f(x_2)$ [1]

$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

After solving

$-x_1 = -x_2$

$\Rightarrow x_1 = x_2$

Hence f is one-one function. [1]

For Onto

$$x = \frac{3y-2}{y-1}$$ [1]

Hence f is onto function.

Thus f is one-one onto function.

If f^{-1} is inverse function of f then

$$f^{-1}(y) = \frac{3y-2}{y-1}$$ [1]

17. $f = R \times R$

$f(x) = x^2 + 2$

$g : R \times R$

$$g(x) = \frac{x}{x-1}, x \neq 1$$

fog $= f(g)x))$

$$= f\left(\frac{x}{x-1}\right) = \left(\frac{x}{x-1}\right)^2 + 2 = \frac{x^2}{(x-1)^2} + 2$$

$$= \frac{x^2 + 2(x-1)^2}{(x-1)^2} = \frac{x^2 + 2x^2 - 4x + 2}{(x-1)^2}$$

$$= \frac{3x^2 - 4x + 2}{(x-1)^2}$$ [1]

gof = g(f(x))

$= g(x^2 + 2)$

$$= \frac{(x^2+2)}{(x^2+2)-1}$$

$$= \frac{x^2+2}{x^2+1} = 1 + \frac{1}{x^2+1}$$ [1]

$$\therefore fog(2) = \frac{3(2)^2 - 4(2) + 2}{(2-1)^2} = 6$$ [1]

$$gof(-3) = 1 + \frac{1}{(-3)^2+1} = \frac{11}{10} = 1\frac{1}{10}$$ [1]

18. Here, $R = \{(a, b) : a \leq b\}$

For reflexivity,

As $a \leq a \forall a \in R$

$\because$ 'R' is reflexive. [1]

For symmetry,

suppose $(a, b) \in R$

$\therefore a \leq b$

Here, it is not necessary that $b \leq a$

$\therefore (b, a) \in R$ [False]

Therefore, 'R' is not symmetric. [2]

for transitivity,

Suppose aRb and bRc

$\therefore a \leq b$ and $b \leq c \Rightarrow a \leq c$

$\therefore$ aRc

So, aRb, bRc $\Rightarrow$ aRc

$\therefore$ 'R' is transitive.

Hence, 'R' is reflexive as well as transitive but not symmetric [1]

OR

Given f : N $\to$ N defined as $f(x) = x^2 + x + 1$

Let $x_1, x_2 \in N$, then

$f(x_1) = f(x_2)$

$$\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$

$$\Rightarrow \left(x_1^2 - x_2^2\right) + \left(x_1 - x_2\right) = 0$$

$$\Rightarrow \left(x_1 - x_2\right)\left[x_1 + x_2 + 1\right] = 0$$

$$\because x_1 + x_2 + 1 \neq 0$$

$$\therefore x_1 - x_2 = 0$$

$x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$ [1½]

Therefore, 'f' is one-one and 'f' is not onto as $(x^2 + x + 1)$ does not attain all natural numbers for $x \in N$. [½]

Hence, 'f' is one-one but not onto.

Now, $f(x) = x^2 + x + 1$

On putting $x = f^{-1}(x)$

$$\therefore x = \left(f^{-1}(x)\right)^2 + f^{-1}(x) + 1$$

$$\Rightarrow \left(f^{-1}(x)\right)^2 + f^{-1}(x) + (1 - x) = 0$$

$$\Rightarrow f^{-1}(x) = \frac{-1 \pm \sqrt{(1)^2 - 4(1-x)}}{2}$$

$$= \frac{-1 \pm \sqrt{1 - 4 + 4x}}{2} = \frac{-1 + \sqrt{4x - 3}}{2}$$

Hence, $f^{-1}(x) = \dfrac{-1 + \sqrt{4x-3}}{2}$ [$\because$ S contains natural numbers only.] [2]

19. As given that,

R = {(x, y) : x – y is divisible by 5}

For, reflexive, let $x \in z$

$\Rightarrow$ x – x = 0, which is divisible by 5.

$\Rightarrow$ $(x, x) \in R$ [1]

So, R is reflexive,

For, symmetric, let $x, y \in z$

$(x, y) \in R \Rightarrow (x - y)$ is divisible by 5.

$\Rightarrow (y - x)$ is also divisible by 5.

$\Rightarrow (y, x) \in R$ [1]

So, R is symmetric

For, transitive, let $x, y, z \in z$

$(x, y) \in R \Rightarrow (x - y)$ is divisible by 5.

And $(y, z) \in R \Rightarrow (y - z)$ is divisible by 5.

Now, $(x - z) = (x - y) + (y - z)$ which is divisible by 5. [2]

So, R is transitive.

Hence, R is equivalence relation.

20. Given $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$

i.e. n(B) = 3 and n(G) = 2

as we know if n(A) = m and n(B) = n

Number of relations A $\to$ B = 2^{mn}

(I) Number of relations B to G

$= 2^{3 \times 2} = 2^6 = 64$ [1]

(II) Number of functions from B to G

$= n^m = 2^3 = 8$ [1]

(III) R : B $\to$ B will be

$R = \{(b_1, b_1), (b_1, b_2), (b_1, b_3), (b_2, b_1), (b_2, b_2), (b_2, b_3), (b_3, b_1), (b_3, b_2), (b_3, b_3)\}$

as b_1, b_2, b_3 are all boys

as $\forall(b_i, b_i)$ are present

$\therefore$ R is reflexive

$\forall(b_i, b_j)$ there exist (b_j, b_i)

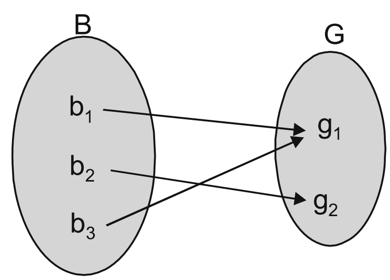
$\therefore$R is symmetric and as $\forall(b_i, b_j), (b_j, b_k)$ there is (b_i, b_k)

$\therefore$ R is transitive [2]

Hence R is an equivalence relation

OR

(III) f : B $\to$ G has mapping diagram as below:



Clearly range of f is (g_1, g_2) = codomain of f.

but element g_1 has two pre–images

$\therefore$f is into but not one–one hence f is not bijec tive. [2]

21. $f(x) = 9x^2 + 6x - 5$.

Let y be an arbitrary element of range f. **[1]**

Then $y = 9x^2 + 6x - 5$

$$\Rightarrow y = 9\left(x^2 + \frac{6}{9}x - \frac{5}{9}\right)$$

$$\Rightarrow y = 9\left[x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \frac{5}{9} - \left(\frac{1}{3}\right)^2\right] \quad \text{[1]}$$

$$\Rightarrow y = 9\left[\left(x + \frac{1}{3}\right)^2 - \frac{6}{9}\right] \quad \text{...(i)}$$

$$\Rightarrow \frac{y}{9} = \left(x + \frac{1}{3}\right)^2 - \frac{6}{9} \Rightarrow \frac{y}{9} + \frac{6}{9} = \left(x + \frac{1}{3}\right)^2$$

Taking square root on both sides

$$\left(x + \frac{1}{3}\right) = \frac{\sqrt{y+6}}{3} \quad \text{[1]}$$

$$x = \frac{\sqrt{y+6}}{3} - \frac{1}{3} = \frac{\sqrt{y+6}-1}{3}$$

Or $g(y) = \dfrac{\sqrt{y+6}-1}{3}$

$$\Rightarrow gof(x) = g(f(x)) = g\left(9x^2 + 6x - 5\right) \quad \text{[1]}$$

$$= \frac{\sqrt{9x^2 + 6x - 5 + 6} - 1}{3}$$

$$= \frac{\sqrt{9x^2 + 6x + 1} - 1}{3}$$

$$= \frac{\sqrt{(3x+1)^2} - 1}{3} = \frac{3x + 1 - 1}{3}$$

$$= \frac{3x}{3} = x \quad \text{...(ii)}$$

And fog $(y) = f(g(y))$

$$= f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$= 9\left[\left(\frac{\sqrt{y+6}-1}{3} + \frac{1}{3}\right)^2 - \frac{6}{9}\right] \text{.... from (i)}$$

$$= 9\left[\left(\frac{\sqrt{y+6}-1+1}{3}\right)^2 - \frac{6}{9}\right]$$

$$= 9\left[\left(\frac{\sqrt{y+6}}{3}\right)^2 - \frac{6}{9}\right]$$

$$= 9\left[\frac{y+6}{9} - \frac{6}{9}\right] = \frac{9(y+6)}{9} - \frac{6 \times 9}{9}$$

$= y + 6 - 6 \ = y$ ...(iii)

From (ii) and (iii), f is invertible and $f^{-1} = g$.

$$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}, y \in \mathrm{S}$$

(i) $f^{-1}(10) = \dfrac{\sqrt{10+6}-1}{3}$

$$\frac{\sqrt{16}-1}{3} = \frac{4-1}{3} = \frac{3}{3} = 1 \quad \text{[1]}$$

(ii) $f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$

$$\therefore \frac{4}{3} = \frac{\sqrt{y+6}-1}{3}$$

$$\therefore 4 + 1 = \sqrt{y+6}$$

$$\therefore 5 = \sqrt{y+6} \quad \text{[1]}$$

Squaring on both sides we get

$25 = y + 6$

$\therefore \ y = 19$

22. (*i*) Reflexive: Clearly $(a, b)\,R\,(a, b)$s

Since $ab(b+a) = ba(a+b) \,\forall (a,b) \in N \times N$ **[1]**

$\therefore$ R is reflexive.

(*ii*) Symmetric: Let $(a,b)R(c,d)$

$$\Rightarrow ad(b+c) = bc(d+a)$$

$$\Rightarrow cb(d+a) = da(c+b) \quad \text{[1]}$$

$$\Rightarrow (c,d)R(a,b) \,\forall (a,b),(c,d) \in N \times N$$

$\therefore$ R is symmetric.

(iii) Let $(a,b)R(c,d)\,and\,(c,d)R(e,f)$

for $a,b,c,d,e,f \in N$ [1]

$\therefore ad(b+c)=bc(a+d)\,and\,cf(d+e)=de(c+f)$

$\frac{b+c}{bc}=\frac{a+d}{ad}$	$\frac{d+e}{de}=\frac{c+f}{cf}$
$\frac{b}{bc}+\frac{c}{bc}=\frac{a}{ad}+\frac{d}{ad}$	$\frac{d}{de}+\frac{e}{de}=\frac{c}{cf}+\frac{f}{cf}$
$\frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a}$.....$(i)$	$\frac{1}{e}+\frac{1}{d}=\frac{1}{f}+\frac{1}{c}$ .....(ii)

[1]

Adding (i) and (ii), we have

$$\frac{1}{c}+\frac{1}{b}+\frac{1}{e}+\frac{1}{d}=\frac{1}{d}+\frac{1}{a}+\frac{1}{f}+\frac{1}{c}$$ [1]

$$\frac{e+b}{be}=\frac{f+a}{af}$$

$$af(b+e)=be(a+f)$$

Hence, $(a,d)R(e,f)$

$\therefore$ R is transitive

$\because$ R is reflexive, symmetric and transitive, [1]
Hence R is an equivalence relation.

23. $f(x)=9x^2+6x-5$

Let y be an arbitrary element of range f. [1]

Then $y=9x^2+6x-5$

$$\Rightarrow y=9\left(x^2+\frac{6}{9}x-\frac{5}{9}\right)$$

$$\Rightarrow y=9\left[x^2+\frac{2}{3}x+\left(\frac{1}{3}\right)^2-\frac{5}{9}-\left(\frac{1}{3}\right)^2\right]$$

$$\Rightarrow y=9\left[\left(x+\frac{1}{3}\right)^2-\frac{6}{9}\right] \quad ...(i)$$ [1]

$$\Rightarrow \frac{y}{9}=\left(x+\frac{1}{3}\right)^2-\frac{6}{9}$$

$$\Rightarrow \frac{y}{9}+\frac{6}{9}=\left(x+\frac{1}{3}\right)^2$$

Taking square root on both sides

$$\left(x+\frac{1}{3}\right)=\frac{\sqrt{y+6}}{3}$$

$$x=\frac{\sqrt{y+6}}{3}-\frac{1}{3}=\frac{\sqrt{y+6}-1}{3}$$

Or $g(y)=\dfrac{\sqrt{y+6}-1}{3}$

$$\Rightarrow gof(x)=g(f(x))=g\left(9x^2+6x-5\right)$$ [1]

$$=\frac{\sqrt{9x^2+6x-5+6}-1}{3}$$

$$=\frac{\sqrt{9x^2+6x+1}-1}{3}$$

$$=\frac{\sqrt{(3x+1)^2}-1}{3}=\frac{3x+1-1}{3}$$

$$=\frac{3x}{3}=x \quad ....(ii)$$

And $fog(y)=f(g(y))$

$$=f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$=9\left[\left(\frac{\sqrt{y+6}-1}{3}+\frac{1}{3}\right)^2-\frac{6}{9}\right] \quad \text{(from (i))}$$

$$=9\left[\left(\frac{\sqrt{y+6}-1+1}{3}\right)^2-\frac{6}{9}\right]$$ [1]

$$=9\left[\left(\frac{\sqrt{y+6}}{3}\right)^2-\frac{6}{9}\right]$$

$$=9\left[\frac{y+6}{9}-\frac{6}{9}\right]$$

$= \frac{9(y+6)}{9} - \frac{6 \times 9}{9}$

$= y + 6 - 6 = y$ ....(iii)

From (ii) and (iii), invertible and $f^{-1} = g$.

$f^{-1}(y) = \frac{\sqrt{y+6-1}}{3}, y \in S$ **[1]**

or $f^{-1}(43) = \frac{\sqrt{43+6}-1}{3}$

$= \frac{7-1}{3} = \frac{6}{3} = 2$

or $f^{-1}(163) = \frac{\sqrt{163+6}-1}{3}$

$= \frac{13-1}{3} = \frac{12}{3} = 4$ **[1]**

24. $f(x) = \frac{4x+3}{3x+4}\ x \in R - \left\{-\frac{4}{3}\right\}$

F is one-one

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$ **[1]**

$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$

$\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12$

$= 12x_1x_2 + 9x_1 + 16x_2 + 12$

$\Rightarrow 7x_1 = 7x_2 \Rightarrow x_1 = x_2$

Thus f is one-one

Now, f is onto

Let $k \in R - \left\{\frac{4}{3}\right\}$ be any number. **[1]**

$f(x) = k$

$\Rightarrow \frac{4x+3}{3x+4} = k$

$\Rightarrow 4x + 3 = 3kx + 4k$

$\Rightarrow x = \frac{4k-3}{4-3k}$

Also $\frac{4k-3}{4-3k} = -\frac{4}{3}$

Implies –9 = – 16 (Which is impossible)

$\therefore f\left(\frac{4k-3}{4-3k}\right) = k$, i.e. f is onto **[1]**

$\therefore$ The function f is invertible. i.e. f^{-1} exist inverse of f.

Let $f^{-1}(x) = k$

$f(k) = x$

$\Rightarrow \frac{4k+3}{3k+4} = x$ **[1]**

$\Rightarrow k = \frac{(4x-3)}{4-3x}$

$\therefore f^{-1}(x) = \frac{4x+3}{4-3x}$,

$x \in R - \left\{-\frac{4}{3}\right\}$

$f^{-1}(0) = -\frac{3}{4}$

When $f^{-1}(x) = 2$ **[1]**

$\Rightarrow \frac{(4x-3)}{4-3x} = 2$

$\Rightarrow 4x - 3 = 8 - 6x$

$\Rightarrow 10x = 11$

$\Rightarrow x = \frac{11}{10}$ **[1]**

25. $A = R \times R$

$(a,b) * (c,d) = (a+c, b+d)$

Commutative :

Lete $(a, b), (c, d) \in A$

$= (c + a, d + b)$

$= (c, d) * (a, b)\ \forall (a, b)(c, d) A$ **[1½]** where * is commutative

Associative:

Let $(a,b),(c,d),(e,f) \in A$

$((a,b)*(c,d))*(e,f) = (a+c, b+d)*(e,f)$

$= (a+c+e, b+d+f)$

$= (a+(c+e), b+(d+f))$

$= (a,b)*(c+e, d+f)$

$= (a,b)*((c,d)(e,f))$

$\forall (a,b),(c,d),(e,f) \in A$ **[1½]**

where * is associative

Identity element:

Let $(e_1, e_2) \in A$

is identity element for * operation by definition

$\Rightarrow (a,b)*(e_1,e_2) = (a,b)$

$\Rightarrow (a+e_1, b+e_2) = (a,b)$

$a+e_1 = a, b+e_2 = b$

$\Rightarrow e_1 = 0, e_2 = 0$

$\Rightarrow (0,0) \in A$

$\Rightarrow$ (0, 0) is identity element for * **[1½]**

Inverse:

Let $(b_1, b_2) \in A$ is inverse of element $(a,b) \in A$

then by definition

$(a,b)*(b_1,b_2) = (0,0)$

$(a+b_1, b+b_2) = (0,0)$

$\Rightarrow a+b_1 = 0, b+b_2 = 0$

$\Rightarrow (-a,-b) \in A$ is inverse of every element $(a, b) \in A$. **[1½]**

26. Given defined by $f(x) = \dfrac{x}{x^2+1}, \forall x \in \mathbb{R}$

Checking for One - One (Injectivity):

Let $a, b \in R$ Then,

$f(a) = f(b)$ **[1]**

$$\Rightarrow \frac{a}{a^2+1} = \frac{b}{b^2+1}$$

$$a(b^2+1) = b(a^2+1)$$

$$ab^2 + a = ba^2 + b$$

$$ab^2 + a - ba^2 - b = 0$$

$$ab(b-a) - 1(b-a = 0)$$

$$b-a = 0 \text{ or } ab-1 = 0$$

$$b = a \text{ or } ab = 1$$

$$\Rightarrow a = \frac{1}{b}$$

Hence, $f(x)$ is not one-one or Injective. **[1]**

Onto (Surjectivity):

Let $y \in R$

Then $y = f(x)$

$$\Rightarrow y = \frac{x}{x^2+1}$$ **[1]**

$x^2 y + y = x$

$x^2 y + y - x = 0$

To solve for x we use the quadratic formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where

$a = y$, $b = -1$ and $c = y$

$$x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$1 - 4y^2 \geq 0$$

$$\Rightarrow 4y^2 \leq 1$$

$$\Rightarrow y^2 \leq \frac{1}{4}$$

$$y^2 \le \left(\frac{1}{2}\right)^2$$

$$y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$\left[x^2 \le a^2 \text{ then } -a \le x \le \text{a}\right]$ [1]

$\Rightarrow$ Range is not equal to codomain.

Thus, $f(x)$ is not onto.

Given $f(x) = \dfrac{x}{x^2+1}$, $g(x) = 2x - 1$

$$fog(x) = f(g(x))$$

$$fog(x) = f(2x-1)$$

$$fog(x) = \frac{2x-1}{(2x-1)^2+1}$$

$$fog(x) = \frac{2x-1}{4x^2+1-4x+1}$$

$$fog(x) = \frac{2x-1}{4x^2-4x+2}$$

$$fog(x) = \frac{2x-1}{2(2x^2-2x+1)}$$ [2]

27. $A = \{x \in \mathbf{Z} : 0 \le x \le 12\}$,

$\{(a, b) : |a-b| \text{ is divisible by } 4\}$

Equivalence relation:

Let $x \in A$

$|x-x| = 0$. and 0 is divisible by 4. [1]

$\Rightarrow |x-x| \in \mathbb{R}$

$\therefore R$ is reflexive.

Let $(x, y) \in A$, $|\text{x}-\text{y}|$ is divisible by 4.

$\Rightarrow |x-y| = 4k, \quad k \in \mathbb{Z}$

$\Rightarrow |y-x| = 4k, \quad k \in \mathbb{Z}$

$\Rightarrow (x, y) \in R$

$\therefore R$ is symmetric. [1]

Transitive:

Let $x, \text{y}, z \in A$, $|\text{x}-\text{y}|$ is divisible by 4.

$\Rightarrow |x-y| = 4k, \quad k \in \mathbb{Z}$ [1]

$\Rightarrow x - y = 4k$ or $x - y = -4k$

$x - y = \pm 4k$ (1)

And assume $|y-z|$ is divisible 4.

$\Rightarrow |y-z| = 4n, \quad n \in \mathbb{Z}$

$\Rightarrow y - z = 4n$ or $\text{y-z} = -4n$

$y - z = \pm 4n$ (2)

Adding (1) and (2) we get.

$$(x-y)+(y-z) = (\pm 4k)+(\pm 4n)$$

$$x - z = \pm 4(k+n)$$

$\Rightarrow x - z$ is divisible by 4.

$|x-z|$ is divisible by 4.

$\Rightarrow (x, z) \in R$

$\therefore$ It is transitive. [1]

As all three properties are satisfied so given relation R is an equivalence relation.

Let a be an element of A such that $(a, 1) \in R$

$|a-1|$ is divisible by 4.

$0 \le a \le 12$

$|a-1| = 0, 4, 8, 12$

$a - 1 = 0, 4, 8, 12$

$a - 1 = 0$

$a = 1$

$a - 1 = 4$

$a = 5$

$a = -1 = 8$

$a = 9$

$a - 1 = 12$

$a = 13$ which is not possible [1]

{1, 5, 9} is the set of elements of A related to 1.

To find the equivalence class [2]

Let a be an element of A such that $(a, 2) \in$ R

$|a-2|$ is divisible by 4.

$|a-2| = 4n \qquad n \in \mathbb{Z}$

$|a-2| = 0, 4, 8, 12$

$a-1 = 0, 4, 8, 12$

$a-2=0$

$a=2$

$a-2=4$

$a=6$

$a-2=8$

$a=10$

$a-2=14$

a =14 which is not possible **[1]**

[2, 6, 10] is the equivalence class of [2].

MULTIPLE CHOICE QUESTIONS

1. Let A = {1, 2, 3}. Then, the relation R = { (1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)} on A is-

(*a*) Reflexive and transitive but not symmetric.

(*b*) Symmetric and transitive but not reflexive.

(*c*) Reflexive and symmetric but not transitive.

(*d*) Reflexive, symmetric & transitive.

2. Let R be a relation on the set N of all natural numbers, defined by a R b $\Leftrightarrow$ a is a factor of b. Then, R is

(*a*) Reflexive and symmetric, but not transitive.

(*b*) Symmetric and transitive, but not reflexive.

(*c*) Reflexive and transitive, but not symmetric.

(*d*) An equivalence relation.

3. If R = {(x, y) : x + 2y = 8} is a relation on N, then, the range of R is

(*a*) {1, 2, 3} (*b*) {1, 2, 3, 4}

(*c*) {1, 2, 3, 4, 5} (*d*) {1}

4. If $R \to R$ is defined by f(x) = 3x + 2, then find f[f(x)]

(*a*) 9x + 6 (*b*) 9x + 8

(*c*) 6x + 8 (*d*) 6x + 9

5. A relation R on A reflexive, only when

(*a*) $R^{-1} = R$

(*b*) $RoR \subseteq R$

(*c*) $I_A \subseteq R$, where I_A is the identity relation on A.

(*d*) None of these

6. A relation R on A is symmetric, if

(*a*) $I_A \subseteq R$ (*b*) $RoR \subseteq R$

(*c*) $R^{-1} = R$ (*d*) None of these

7. A relation R on a set A is transitive, if

(*a*) $R^{-1} = R$ (*b*) $I_A \subseteq R$

(*c*) $RoR \subseteq R$ (*d*) None of these

8. Let $f : R \to R : f(x) = x^3$, then f is

(*a*) One - one, onto (*b*) One - one, into

(*c*) Many - one, onto (*d*) Many - one, into

9. If $f : Q \to Q : f(x) = 3x + 5$, then $f^{-1}(x) = ?$

(*a*) $3x - 5$ (*b*) $\frac{1}{(3x-5)}$

(*c*) $\frac{1}{3}(x-5)$ (*d*) None of these

10. If $f(x) = (x^2 - 1)$ and $g(x) = (3x + 1)$, then $(gof)(x) = ?$

(*a*) $9x^2 + 6x$

(*b*) $3x^2 - 1$

(*c*) $2x^2 - 1$

(*d*) $3x^2 - 2$

11. If the function $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$

(*a*) $\sqrt[3]{\frac{y-7}{2}}$ (*b*) $\sqrt[3]{\frac{2}{y-7}}$

(*c*) $\sqrt{\frac{y-7}{7}}$ (*d*) None of these

12. If $f(x) = [x]$ and $g(x) = |x|$, then $(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right) = ?$

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) $\frac{1}{2}$

13. If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \neq 0$, then $f(x) = ?$

(*a*) x^2 (*b*) $(x^2 - 1)$

(*c*) $(x^2 - 2)$ (*d*) None of these

14. If f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}, then gof = ?

(*a*) {(1, 3), (3, 1), (4, 3)}

(*b*) {(1, 5), (2, 5), (5, 2)}

(*c*) {(3, 1), (1, 3), (3, 4)}

(*d*) {(5, 1), (5, 2), (2, 5)}

15. Let $f(x) = \frac{x}{\left(x^2 - 3x + 2\right)}$, then Dom (f) = ?

(*a*) R

(*b*) R – {1}

(*c*) R – {1, 2}

(*d*) None of these

Answer Keys

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*b*)	**5.** (*c*)
6. (*c*)	**7.** (*c*)	**8.** (*a*)	**9.** (*c*)	**10.** (*d*)
11. (*a*)	**12.** (*b*)	**13.** (*c*)	**14.** (*a*)	**15.** (*c*)

Solutions

1. Since (1, 1), (2, 2), (3, 3) are in R,

$\therefore$ R is reflexive.

Also, $(a, b) \in R \Rightarrow (b, a) \in R$.

$\therefore$ R is symmetric.

But $(3, 2) \in R$, $(2, 1) \in R$, while $(3, 1) \notin R$.

$\therefore$ R is not transitive. **[1]**

2. $a \mid a \Rightarrow$ R is is reflexive.

$2 \mid 6$ but 6 is not a factor of 2.

$a \mid b, b \mid c \Rightarrow c = bm, b = an$ for some $m, n \in N$

$\Rightarrow c = amn$ for some $m, n \in N \Rightarrow a \mid c$

$\therefore$ R is reflexive and transitive but not symmetric. **[1]**

3. R = {(x, y) : x + 2y = 8} is a relation on N Then, we can say, 2y = 8 – x

$$\Rightarrow y = \frac{8 - x}{2} = 4 - \frac{x}{2}$$

So, we can put the value of x, x = 2, 4, 6 only we get y = 3 at x = 2,

y = 2 at x = 4

and y = 1 at x = 6

Hence, required range = {1, 2, 3} **[1]**

4. According to the question,

f(x) = 3x + 2

Now, f[f(x)] = 3(3x + 2) + 2

= 9x + 6 + 2

= 9x + 8 **[1]**

5. A relation R on A is reflexive only when $I_A \subseteq R$. **[1]**

6. A relation R on A is symmetric only when $R^{-1} = R$. **[1]**

7. A relation R on A is transitive, if $RoR \subseteq R$. **[1]**

8. $f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one - one.

for each $x \in R$ there exists

$$x^{\frac{1}{3}} \in R \Rightarrow f\left(x^{\frac{1}{3}}\right) = \left(x^{\frac{1}{3}}\right)^3 = x.$$

$\therefore$ f is onto.

Hence, f is one-one and onto. **[1]**

9. Let f(x) = y, then

$x = f^{-1}(y)$

$$y = 3x + 5 \Rightarrow x = \frac{1}{3}(y - 5)$$

$$\Rightarrow f^{-1}(y) = \frac{1}{3}(y - 5)$$

$$\therefore f^{-1}(x) = \frac{1}{3}(x - 5)$$ **[1]**

10. $\because$ $f(x) = (x^2 - 1)$ and $g(x) = (3x + 1)$

$\therefore$ (gof) (x) = g[f(x)]

$= g[x^2 - 1]$

$= 3(x^2 - 1) + 1$

$= 3x^2 - 3 + 1$

$= 3x^2 - 2$ **[1]**

11. Let y = (fog) (x) and y = h(x)

$\therefore$ $y = f[g(x)] = f[x^3 + 5]$

$= 2x^3 + 7$

$$\Rightarrow x = \sqrt[3]{\frac{y - 7}{2}} = h^{-1}(y)$$

$$\Rightarrow (fog)^{-1}(x) = \sqrt[3]{\frac{y - 7}{2}}$$ **[1]**

12. $(gof)\left(-\frac{5}{3}\right) = g\left[f\left(-\frac{5}{3}\right)\right]$

$= g\left[-2+\frac{1}{3}\right] = g(-2) = |-2| = 2$

and $(fog)\left(-\frac{5}{3}\right) = f\left[g\left(-\frac{5}{3}\right)\right] = f\left(\left|-\frac{5}{3}\right|\right)$

$= f\left(\frac{5}{3}\right) = f\left(1+\frac{2}{3}\right) = \left[1+\frac{2}{3}\right] = 1$

$\therefore$ Given expression = 2 – 1 = 1 **[1]**

13. Let $x+\frac{1}{x} = z$, then

$f(z) = f\left(x+\frac{1}{x}\right) = \left(x^2+\frac{1}{x^2}\right)$

$= \left(x+\frac{1}{x}\right)^2 - 2 = \left(z^2-2\right)$

$\therefore$ $f(x) = (x^2 - 2)$ **[1]**

14. Dom (gof) = Dom (f)

$\therefore$ (gof) (1) = f(f(1)] = g(2) = 3

(gof) (3) = f(f(3)] = g(5) = 1

(gof) (4) = f[f(4)] = g(1) = 3

$\therefore$ gof = {(1,3), (3, 1), (4, 3)} **[1]**

15. Given $f(x) = \frac{x}{(x-1)(x-2)}$

Clearly, f(x) is not defined when

x – 1 = 0 or x – 2 = 0

i.e. when x = 1 or x = 2

$\therefore$ Dom (f) = R – {1, 2}. **[1]**