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2025

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Unit V Machine Design

Unit VI Fluid Mechanics and Hydraulic Machinery

3 CHAPTER

Fluid Kinematics

Fluid Velocity

1. The velocity components in the x and y directions of a two dimensional potential flow are u and v, respectively. Then $\frac{\partial u}{\partial x}$, is equal to

(a) $\frac{\partial v}{\partial x}$ (b) $-\frac{\partial v}{\partial x}$

(c) $\frac{\partial v}{\partial y}$ (d) $-\frac{\partial v}{\partial y}$

[2005 : 1 Mark]

2. A leaf is caught in a whirlpool. At a given instant, the leaf is at a distance of 120 m from the centre of the whirlpool. The whirlpool can be described by the following velocity distribution: $V_r = -\left(\frac{60\times10^3}{2\pi r}\right)$ m/s and $V_\theta = \frac{300\times10^3}{2\pi r}$ m/s, where r (in meters) is the distance from the centre of the whirlpool. What will be the distance of the leaf from the centre when it has moved through half a revolution?

(a) 48 m (b) 64 m

(c) 120 m (d) 142 m

[2005 : 2 Marks]

3. In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1 + 3x/L)i$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is

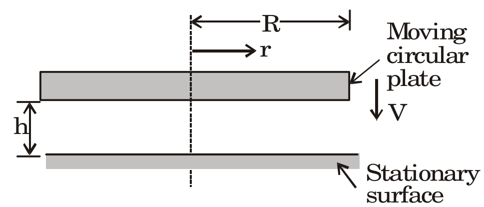
(a) $\frac{L}{u_0}$ (b) $\frac{L}{3u_0}\ln 4$

(c) $\frac{L}{4u_0}$ (d) $\frac{L}{2.5u_0}$

[2007 : 1 Mark]

Linked Answer Questions 4 and 5:

The gap between a moving circular plate and a stationary surface is being continuously reduced, as the circular plate comes down at a uniform speed V towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid.



4. The radial velocity V_r at any radius r, when the gap width is h, is

(a) $V_r = \frac{Vr}{2h}$ (b) $V_r = \frac{2Vr}{h}$

(c) $V_r = \frac{2Vh}{r}$ (d) $V_r = \frac{Vh}{r}$

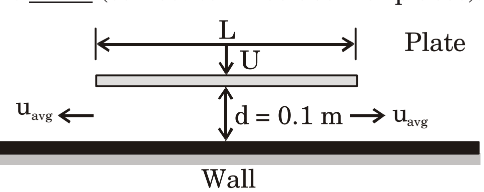
[2008 : 2 Marks]

5. The radial component of the fluid acceleration at r = R is

(a) $\frac{3V^2R}{4h^2}$ (b) $\frac{V^2R}{4h^2}$

(c) $\frac{V^2R}{2h^2}$ (d) $\frac{V^2h}{2R^2}$ **[2008 : 2 Marks]**

6. A flat plate of width L = 1 m is pushed down with a velocity U = 0.01 m/s towards a wall resulting in the drainage of the fluid between the plate and the wall as shown in the figure. Assume two- dimensional incompressible flow and that the plate remains parallel to the wall. The average velocity, u_{avg} of the fluid (in m/s) draining out at the instant shown in the figure is _____ (correct to three decimal places).



[2018 : 1 Mark, Set-1]

7. The velocity field of an incompressible flow in a Cartesian system is represented by

$$\vec{V} = 2(x^2 - y^2)\hat{i} + v\hat{j} + 3\hat{k}$$

Which one of the following expressions for v is valid?

(a) $-4xz + 6xy$ (b) $-4xy - 4xz$

(c) $4xz - 6xy$ (d) $4xy + 4xz$

[2020 : 1 Mark, Set-1]

8. A two dimensional flow has velocities in x and y directions given by $u = 2xyt$ and $v = -y^2t$, where t denotes time. The equation for streamline passing through $x = 1, y = 1$ is

(a) $x^2y = 1$ (b) $xy^2 = 1$

(c) $x^2y^2 = 1$ (d) $x/y^2 = 1$

[2021 : 1 Mark, Set-2]

9. The velocity field in a fluid is given to be $\vec{V} = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$. Which of the following statement(s) is/are correct?

(a) The velocity field is one-dimensional.

(b) The flow is incompressible.

(c) The flow is irrotational.

(d) The acceleration experienced by a fluid particle is zero at (x = 0, y = 0).

[2022 : 1 Mark, Set-2]

10. The velocity field of a certain two-dimensional flow is given by

$$V(x, y) = k(x\hat{i} - y\hat{j})$$

where k = 2 s^{-1}. The coordinates x and y are in meters. Assume gravitational effects to be negligible.

If the density of the fluid is 1000 kg/m3 and the pressure at the origin is 100 kPa, the pressure at the location (2m, 2m) is ______ kPa.

(Answer in integer)

[2023 : 2 Marks]

11. Consider a unidirectional fluid flow with the velocity field given by

$V(x, y, z, t) = U(x, t)\hat{i}$

where u(0, t) = 1.

If the spatially homogeneous density field varies with time t as $\rho(t) = 1 + 0.2\,e^{-t}$

the value of u(2, 1) is. (Rounded off to two decimal places)

Assume all quantities to be dimensionless.

[2023 : 2 Marks]

Fluid Acceleration

12. In a two-dimensional velocity field with velocities u and v along the x and y directions respectively, the convective acceleration along the x-direction is given by

(a) $u\dfrac{\partial V}{\partial x} + v\dfrac{\partial u}{\partial y}$ (b) $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial v}{\partial y}$

(c) $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}$ (d) $u\dfrac{\partial u}{\partial x} + u\dfrac{\partial u}{\partial y}$

[2006 : 1 Mark]

13. A flow field which has only convective acceleration is

(a) a steady uniform flow

(b) an unsteady uniform flow

(c) a steady non-uniform flow

(d) an unsteady non-uniform flow

[2014 : 1 Mark, Set-4]

14. Consider the two-dimensional velocity field given by $\bar{V} = (5 + a_1x + +b_1y)\hat{i} + (4 + a_2x + b_2y)\hat{j}$, where a_1, b_1, a_2 and b_2 are constants. Which one of the following conditions needs to be satisfied for the flow to be incompressible?

(a) $a_1 + b_1 = 0$ (b) $a_1 + b_2 = 0$

(c) $a_2 + b_2 = 0$ (d) $a_2 + b_1 = 0$

[2017 : 1 Mark, Set-1]

15. Water flows though a pipe with a velocity given by $\vec{V} = \left(\dfrac{4}{t} + x + y\right)\hat{j}$ m/s, where $\hat{j}$ is the unit vector in the y direction, t(>0) is in seconds, and x and y are in meters. The magnitude of total acceleration at the point (x, y) = (1, 1) at t = 2 s is _____ m/s^2.

[2019 : 1 Mark, Set-1]

16. For a two-dimensional, incompressible flow having velocity components u and v in the x and y directions, respectively, the expression

$$\frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y}$$

can be simplified to

(a) $u\dfrac{\partial u}{\partial x} + u\dfrac{\partial v}{\partial y}$ (b) $2u\dfrac{\partial u}{\partial x} + u\dfrac{\partial v}{\partial y}$

(c) $2u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}$ (d) $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}$

[2021 : 1 Mark, Set-2]

17. A steady two-dimensional flow field is specified by the stream function

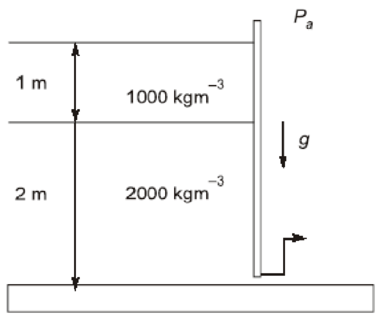
$\Psi = kx^3y$,

where x and y are in meter and the constant $k = 1\ m^{-2}s^{-1}$. The magnitude of acceleration at a point (x, y) = (1 m, 1 m) is _______ m/s^2 (round off to 2 decimal places).

[2022 : 2 Marks, Set-1]

18. The figure shows two fluids held by a hinged gate. The atmospheric pressure is Pa = 100 kPa. The moment per unit width about the base of the hinge is ________ kNm/m. (Rounded off to one decimal place)

Take the acceleration due to gravity to be $g = 9.8\ m/s^2$.



[2023 : 2 Marks]

Types of Fluid Flow

19. The 2-D flow with, velocity

$\vec{V} = (x + 2y + 2)\hat{i} + (4-y)\hat{j}$ is

(a) Compressible and irrotational
(b) Compressible and not irrotational
(c) Incompressible and irrotational
(d) Incompressible and not irrotational

[2001 : 2 Marks]

20. Which combination of the following statements about steady incompressible forced vortex flow is correct?

P : Shear stress is zero at all points in the flow.
Q : Vorticity is zero at all points in the flow.
R : Velocity is directly proportional to the radius from the centre of the vortex.
S : Total mechanical energy per unit mass is constant in the entire flow field. Select the correct answer using the codes given below:

(a) P and Q (b) R and S
(c) P and R (d) P and S

[2007 : 2 Marks]

21. For the continuity equation given by $\vec{\Delta}.\ \vec{V} = 0$ to be valid, where $\vec{V}$ is the velocity vector, which one of the following is a necessary condition?

(a) Steady flow
(b) Irrotational flow
(c) Inviscid flow
(d) Incompressible flow **[2008 : 1 Mark]**

22. For an incompressible flow field, V, which one of the following conditions must be satisfied?

(a) $\nabla . \vec{V} = 0$

(b) $\nabla \times \vec{V} = 0$

(c) $(\vec{V} . \nabla)\vec{V} = 0$

(d) $\dfrac{\partial \vec{V}}{\partial t} + (\vec{V} . \nabla)\vec{V} = 0$ **[2014 : 1 Mark, Set-2]**

23. Match the following pairs:

	Equation		Physical Interpretation
P.	$\nabla \times \vec{V} = 0$	I.	Incompressible continuity equation
Q.	$\nabla.\vec{V} = 0$	II.	Steady flow
R.	$\dfrac{\Delta\vec{V}}{\Delta t} = 0$	III.	Irrotational flow
S.	$\dfrac{\partial\vec{V}}{\partial t} = 0$	IV.	Zero acceleration of fluid particle

(a) P-IV, Q-I, R-II, S-III
(b) P-IV, Q-III, R-I, S-II
(c) P-III, Q-I, R-IV, S-II
(d) P-III, Q-I, R-II, S-IV

[2015 : 2 Marks, Set-1]

24. For a two-dimensional incompressible flow field given by $\vec{u} = A(x\hat{i} - y\hat{j})$, where A > 0, which one of the following statements is FALSE?

A. It satisfies continuity equation
B. It is unidirectional when $x \to 0$ and $y \to \infty$.
C. Its streamlines are given by x = y.
D. It is irrotational

(a) A (b) B
(c) C (d) D

[2018 : 1 Mark, Set-1]

25. In a Lagrangian system, the position of a fluid particle in a flow is described as $x = x_0e^{-kt}$ and $y = y_0e^{kt}$ where t is the time while x_0, y_0, and k are constants. The flow is

(a) unsteady and one-dimensional
(b) steady and two-dimensional
(c) steady and one-dimensional
(d) unsteady and two-dimensional

[2018 : 2 Marks, Set-1]

26. The velocity field of a two dimensional incompressible flow is given by $\vec{V} = 2\sinh x\hat{i} + v(x, y)\hat{j}$, where $\hat{i}$ and $\hat{j}$ denote the unit vectors in x and y direction, respectively. If v (x, 0) = cosh x, then v (0, –1) is

(a) 1 (b) 2
(c) 3 (d) 4

[2024 : 1 Mark]

Fluid Flow Lines and Application

27. A fluid flow is represented by the velocity field $\vec{V} = ax\hat{i} + ay\hat{j}$, where a is a constant. The equation of stream line passing through a point (1, 2) is

(a) $x - 2y = 0$ (b) $2x + y = 0$
(c) $2x - y = 0$ (d) $x + 2y = 0$

[2004 : 1 Mark]

28. A two-dimensional flow field has velocities along the x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. The equation of streamline is

(a) x^2y = constant
(b) xy^2 = constant
(c) xy = constant
(d) not possible to determine **2006 : 2 Marks]**

29. Velocity vector of a flow field is given as $\vec{V} = 2xy\hat{i} - x^2z\hat{j}$. The vorticity vector at (1, 1, 1) is

(a) $4\hat{i} - \hat{j}$ (b) $4\hat{i} - \hat{k}$
(c) $\hat{i} - 4\hat{j}$ (d) $\hat{i} - 4\hat{k}$

[2010 : 2 Marks]

30. A streamline and an equipotential line in a flow field

(a) are parallel to each other
(b) are perpendicular to each other
(c) intersect at an acute angle
(d) are identical

[2011 : 1 Mark]

31. Consider the following statements regarding streamline(s):

(i) It is a continuous line such that the tangent at any point on it shows the velocity vector at that point
(ii) There is no flow across streamlines
(iii) $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$ is the differential equation of a streamline, where u, v and w are velocities in directions x, y and z, respectively
(iv) In an unsteady flow, the path of a particle is a streamline

Which one of the following combinations of the statements is true?

(a) (i), (ii), (iv) (b) (ii), (iii), (iv)
(c) (i), (iii), (iv) (d) (i), (ii), (iii)

[2014 : 2 Marks, Set-4]

32. Consider a velocity field $\breve{V} = K(y\hat{i} + x\hat{k})$, where K is a constant. The vorticity, Ω_Z, is

(a) –K (b) K
(c) –K/2 (d) K/2

[2014 : 2 Marks, Set-4]

33. If the fluid velocity for a potential flow is given by V(x, y) = u(x, y)i + ν(x, y)j with usual notations, then the slope of the potential line at (x, y) is

(a) $\frac{\nu}{u}$ (b) $-\frac{u^2}{\nu}$
(c) $-\frac{\nu}{u^2}$ (d) $\frac{u}{\nu}$

[2015 : 1 Mark, Set-2]

34. The velocity field of an incompressible flow is given by

$V = (a_1x + a_2y + a_3z)i + (b_1x + b_2y + b_3z)j + (c_1x + c_2y + c_3z)k$, where $a_1 = 2$ and $c_3 = -4$. The value of b_2 is _________.

[2015 : 2 Marks, Set-1]

35. The volumetric flow rate (per unit depth) between two streamlines having stream function ψ_1 and ψ_2 is

(a) $|\psi_1 + \psi_2|$ (b) $\psi_1\psi_1$
(c) ψ_1/ψ_2 (d) $|\psi_1 - \psi_2|$

[2016 : 1 Mark, Set-2]

36. For a certain two-dimensional incompressible flow, velocity field is given by $2xy\,\hat{i} - y^2\,\hat{j}$. The streamlines for this flow are given by the family of curves

(a) x^2y^2 = constant

(b) xy^2 = constant

(c) $2xy - y^2$ = constant

(d) xy – constant

[2016 : 1 Mark, Set-3]

37. For a two-dimensional flow, the velocity field is

$\vec{u} = \frac{x}{x^2+y^2}\hat{i} + \frac{y}{x^2+y^2}\hat{j}$ where $\hat{i}$ and $\hat{j}$ are the basis vectors in the $x - y$ Cartesian coordinate system. Identify the CORRECT statements from below.

1. The flow is incompressible
2. The flow is unsteady
3. y-component of acceleration,

$$a_y = \frac{-y}{(x^2+y^2)^2}$$

4. x-component of acceleration,

$$a_x = \frac{-(x+y)}{(x^2+y^2)^2}$$

(a) 2 and 3 (b) 1 and 3

(c) 1 and 2 (d) 3 and 4

[2016 : 2 Marks, Set-3]

38. For a steady flow, the velocity field is $\bar{V} = (-x^2 + 3y)\,\hat{i} + (2xy)\,\hat{j}$. The magnitude of the acceleration of a particle at (1, - 1) is

(a) 2 (b) 1

(c) $2\sqrt{5}$ (d) 0

[2017: 2 Marks, Set-1]

39. A two-dimensional incompressible frictionless flow field is given by $\vec{\mu} = x\hat{i} - y\hat{j}$. If ρ is the density of the fluid, the expression for pressure gradient vector at any point in the flow field is given as

(a) $\rho\left(x\hat{i} - y\hat{j}\right)$ (b) $\rho\left(x\hat{i} + y\hat{j}\right)$

(c) $-\rho\left(x^2\hat{i} + y^2\hat{j}\right)$ (d) $-\rho\left(x\hat{i} + y\hat{j}\right)$

[2019 : 1 Mark, Set-2]

ANSWERS

1. (d) **2.** (b) **3.** (b) **4.** (a) **5.** (a) **6.** (0.05) **7.** (b) **8.** (*b*) **9.** (b,c,d) **10.** (84)
11. (1.137) **12.** (c) **13.** (c) **14.** (b) **15.** (*) **16.** (d) **17.** (4.20 to 4.28) **18.** (57.225)
19. (d) **20.** (b) **21.** (d) **22.** (a) **23.** (c) **24.** (c) **25.** (b) **26.** (c) **27.** (c) **28.** (a)
29. (d) **30.** (b) **31.** (d) **32.** (a) **33.** (b) **34.** (2) **35.** (d) **36.** (b) **37.** (b) **38.** (c)
39. (d)

* Represents Numerical & Subjective Questions.

EXPLANATIONS

1. $\dfrac{\partial^2\phi}{\partial x^2}+\dfrac{\partial^2\phi}{\partial y^2}=0$...(*i*) (Laplace Equation should be satisfied)

$\therefore\ u=-\dfrac{\partial\phi}{\partial x}$ and $v=-\dfrac{\partial\phi}{\partial y}$

$\Rightarrow \dfrac{\partial u}{\partial x}=-\dfrac{\partial^2\phi}{\partial x^2}$, and $\dfrac{\partial v}{\partial y}=-\dfrac{\partial^2\phi}{\partial y^2}$

Putting values in equation (i), we get

$$\left(-\frac{\partial u}{\partial x}\right)+\left(-\frac{\partial v}{\partial y}\right)=0$$

$$\Rightarrow \frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$$

2. Radial distance = 120 m

$$V_r=\frac{-60\times10^3}{2\pi r}\,m/s$$

$$\&\ V_\theta=\frac{300\times10^3}{2\pi r}\,m/s$$

$\&\ \theta=\pi$

$$\frac{V_R}{V_\theta}=\frac{-1}{5}$$

we know that

$$V_r=\frac{dr}{dt}\ ...(i)$$

$$\&\ V_\theta=rw=r\frac{d\theta}{dt}\quad -5V_r=r\frac{d\theta}{dt}$$

$$V_r=\frac{-r}{5}\frac{d\theta}{dt}\ ...(ii)$$

By equating (i) & (ii), we get

$$\frac{dr}{dt}=\frac{-r}{5}\frac{d\theta}{dt}$$

Integrating both sides, we get

$$\int_{120}^{r}\frac{dr}{r}=\frac{-1}{5}[\theta]_0^\pi$$

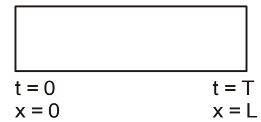
$$\ln r]_{120}^{r}=\frac{-1}{5}\times\pi$$

$$\ln\left(\frac{r}{120}\right)=\frac{-\pi}{5}$$

By solving above, we get

r = 64 m

3.



$$v=u_0\left(1+\frac{3x}{L}\right)$$

$$\therefore\ \frac{dx}{dt}=u_0\left(1+\frac{3x}{L}\right)$$

$$\Rightarrow \frac{dx}{u_0\left(1+\frac{3x}{L}\right)}=dt$$

$$\text{Integrating}\int_0^L\frac{dx}{u_0\left(1+\frac{3x}{L}\right)}=\int_0^T dt$$

$$\Rightarrow \frac{L}{3u_0}\left[\ln\left(1+\frac{3x}{L}\right)\right]_0^L=T$$

$$\Rightarrow T=\frac{L}{3u_0}\ln(4)=\frac{L}{3u_0}\ln(4)$$

4. $V\times\pi r^2=V_r\times2\pi rh$

$$V_r=\frac{v.r}{2h}$$

5. Radial acceleration

$$a_r=V_r\times\frac{\partial Vr}{\partial r}+\frac{\partial V_r}{\partial t}$$

$$a_r=\frac{V.r}{2h}\times\frac{\partial}{\partial r}\left(\frac{Vr}{2h}\right)+\frac{\partial}{\partial t}\left(\frac{Vr}{\partial t}\right)$$

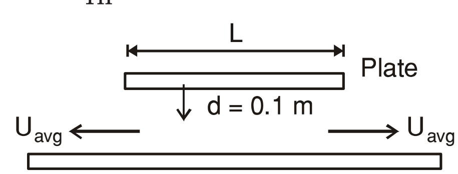
$$\frac{-\partial h}{\partial t}=V$$

$$\therefore a_r=\frac{Vr}{2h}\times\frac{Vr}{2h}+\frac{Vr}{2}\times\left(\frac{-1}{h^2}\frac{\partial h}{\partial t}\right)$$

$$\therefore a_r=\frac{V^2r}{4h^2}+\frac{2V^2r}{4h^2}$$

$$a_r=\frac{3V^2r}{4h^2}$$

6.



By mass conservation:-

$2(s\times u_{avg}\times db)=(s.\,u\times Lb)$

$\Rightarrow\ 2\times v_{avg}\times0.1=0.01\times1$

$\boxed{u_{avg}=0.05\text{ m/s}}$

7. Given : $\vec{V} = 2(x^2 - y^2)\hat{i} + v\hat{j} + 3\hat{k}$
$= u\hat{i} + v\hat{j} + w\hat{k}$

For incompressible flow,

$$\Delta \cdot \vec{V} = 0$$

$$\frac{\partial}{\partial x}(u) + \frac{\partial}{\partial y}(v) + \frac{\partial}{\partial z}(w) = 0$$

$$\Rightarrow \frac{\partial}{\partial x}[2(x^2 - y^2)] + \frac{\partial}{\partial y}(v) + \frac{\partial}{\partial z}(3) = 0$$

$$\Rightarrow 4x + \frac{\partial v}{\partial y} + 0 = 0$$

$$\Rightarrow \frac{\partial v}{\partial y} = -4x$$

$$\Rightarrow v = \int -4x\, dy + \text{cons tan t}$$

$$\Rightarrow v = -4xy + \text{constant}$$

8. $u = 2xyt, \qquad v = -y^2t$

Equation for steam line is

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{2xyt} = \frac{dy}{-y^2t}$$

$$\frac{dx}{2xy} = \frac{-dy}{y^2} \Rightarrow \frac{dx}{2x} = \frac{-dy}{y}$$

Integrating on both sides

$$\frac{1}{2}\log x = -\log y + c$$

at $x = 1$, $y = 1$

$$\frac{1}{2}(\log 1) = -\log(1) + c$$

$c = 0$

$\therefore$ equation of stream line is

$$\frac{1}{2}\log x = -\log y$$

$$\log^{\sqrt{x}} = -\log y \Rightarrow \log^{\sqrt{x}} + \log y = 0$$

$$\sqrt{xy} = 1$$

$xy^2 = 1$

Hence, option (b) is correct answer.

9. As given that :

$$\nabla = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$$

$U = 4xy$, $V = 2(x^2 - y^2)$

(i) For incompressible flow,

$$\vec{\nabla}\cdot\vec{V} = 0$$

or, $$\nabla\cdot\vec{V} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 4y - 4y = 0$$

So, fluid is incompressible.

(ii) For irrotational flow,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(4x - 4x) = 0$$

$(\therefore\ \omega_z = 0)$

So, fluid is irrotational.

(iii) Acceleration at $x = 0$, $y = 0$

$$a_x = \frac{\Delta u}{\Delta t} = \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]_{(x=0,y=0)} = 0$$

$$a_y = \frac{\Delta v}{\Delta t} = \left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]_{(x=0,y=0)} = 0$$

So, acceleration of fluid at $x = 0$, $y = 0$ is zero.

Hence, it is a 2-D flow.

10. As given that,

$$v(x, y) = k\left(x\hat{i} - y\hat{j}\right)$$

$$\rho = 1000 \text{ kg/m}^3$$

$$P_1 = 100 \text{ kPa}$$

At origin (0, 0)

$$V = k\left(x\hat{i} - y\hat{j}\right)$$

$$V_1 = |V_1| = |2(0-0)| = 0$$

At location (2, 2)

$$V_2 = 2\left(2\hat{i} - 2\hat{j}\right)$$

$$V_2 = \left(4\hat{i} - 4\hat{j}\right)$$

$$|V_2| = \sqrt{4^2 + 4^2} = \sqrt{32}$$

Now from Bernoulli's equation;

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

By putting all values;

$$100\text{kPa} + \frac{1}{2}\times 1000\times 0 = P_2 + \frac{1}{2}\times 1000\times\left(\sqrt{32}\right)^2$$

So, $P_2 + 16000 = 100 \times 10^3$

$P_2 = 100000 - 16000$

$= 84{,}000$ pa

$$\boxed{P_2 = 84 \text{ kPa}}$$

11. As given that :

$$u(0, t) = 1$$
$$\rho(t) = 1 + 0.2e^{-t}$$

As we know that,

Continuity equation for unsteady flow :

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z} = 0$$

Here, $V(t, x, y, z) = u(t, x)\,\hat{i}$

Thus, $V = 0$, $w = 0$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho U) = 0 \qquad \text{...(i)}$$

Where, $\frac{\partial \rho}{\partial t} = \frac{\partial\left(1+0.2e^{-t}\right)}{\partial t} = -0.2\ e{-}t$

$$\frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial x}\left(\left(1+0.2e^{-t}\right)u(x,t)\right)$$
$$= \left(1+0.2e^{-t}\right)\frac{\partial u}{\partial x}$$

By putting the value of these expression in equation (i)

$$-\ 0.2\ e^{-t} + \left(1+0.2e^{-t}\right)\frac{\partial u}{\partial x} = 0$$

$$-0.2e^{-t} + \left(1+0.2e^{-t}\right)\cdot\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} = \left[\frac{0.2e^{-t}}{1+0.2e^{-t}}\right]$$

$$u = \left(\frac{0.2e^{-t}}{1+0.2e^{-t}}\right)x + c$$

Using boundary condition,

Since, $u(0, t) = 1$

$$c = 1$$

So, $$u = \left[\frac{0.2e^{-t}}{1+0.2e^{-t}}\right]x + 1$$

And, the value of $u(2, 1) = \left[\frac{0.2e^{-1}}{1+0.2e^{-1}}\right]2 + 1$

$$u(2, 1) = 1.1371 \text{ m/s}$$

$$\boxed{u = 1.14 \text{ m/sec}}$$

12. Two dimensional velocity field with velocities u, v and along x and y direction.

$\therefore$ Acceleration along x direction, $a_x = a_{\text{convective}} + a_{\text{temporal or local}}$

$$= \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\substack{\text{Convective}\\ \text{acceleration}}} + \underbrace{\frac{\partial u}{\partial t}}_{\substack{\text{temporal}\\ \text{acceleration}}}$$

Sicne, $\frac{\partial u}{\partial z} = 0$ for 2-dimensional field, therefore

Convective acceleration

$$= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

13. Convective acceleration is the effect of time independent acceleration of fluid with respect to space that means flow is steady non-uniform flow.

14. For continuous and incompressible flow

$u_x + u_y = 0$

$a_1 + b_2 = 0$

15. Given: $\bar{V} = \left(\frac{4}{t} + x + y\right)\hat{j}\ \frac{m}{s}$

$$\bar{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$u = 0$, $w = 0$

$$V = \left(\frac{4}{t} + x + y\right)\hat{j}$$

$$\vec{a} = \vec{a}_x\hat{i} + \vec{a}_y\hat{j} + \vec{a}_z\hat{k}$$

$$\vec{a}_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} = 0$$

$$\vec{a}_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} = 0$$

$$\vec{a}_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$= v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

$$= \left(\frac{4}{t} + x + y\right) \times 1 + \left(-\frac{4}{t^2}\right)$$

$$\Rightarrow \qquad \vec{a}_y = \left(\frac{4}{t} + x + y - \frac{4}{t^2}\right)\hat{j}$$

Now, At (x,y) = 1,1 and t = 2 sec.

Total acceleration is given by,

$$\vec{a} = \vec{a}_y = \left(\frac{4}{2} + 1 + 1 - \frac{4}{4}\right) = 3\frac{m}{s^2}$$

16. As we know that,

For a two-dimensional compressible flow, continuity equation is

$$\frac{\partial u}{\partial^2 x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y} \Rightarrow \frac{\partial v}{\partial y} = \frac{-\partial u}{\partial x}$$

As given that,

$$\frac{\partial(u^2)}{\partial x}+\frac{\partial(uv)}{\partial y}=2u\frac{\partial u}{\partial x}+\left[\frac{\partial u}{\partial y}(v)+(u)\frac{\partial v}{\partial y}\right]$$

$$=2u\frac{\partial u}{\partial x}+(v)\left(\frac{\partial u}{\partial y}\right)+(u)\left(\frac{\partial v}{\partial y}\right)$$

$$=2u\frac{\partial u}{\partial x}+(V)\left(\frac{\partial u}{\partial y}\right)+(u)\left(\frac{-\partial u}{\partial x}\right)$$

$$=(2u-u)\left(\frac{\partial u}{\partial x}\right)+v\frac{\partial u}{\partial y}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$$

Hence option (d) is the correct answer.

17. As given, Stream function;

$\Psi = kx^3y$; $\quad \therefore k = 1\ m^{-2}s^{-1}$

$$u=\frac{\partial\psi}{\partial y}\Rightarrow u=x^3$$

$$v=\frac{-\partial\psi}{\partial x}\Rightarrow v=-3x^2y$$

$$\vec{V}=x^3\hat{i}-3x^2y\hat{j}$$

$$a_x=u\frac{\delta u}{\delta x}+U\frac{\delta u}{\delta y}$$

$a_x = +x^3(+3x^2) \Rightarrow a_x = 3x^5$

At (1, 1), $a_x = 3\ m/sec^2$

$$a_y=\left[u\frac{\delta u}{\delta x}+v\frac{\delta v}{\delta y}\right]$$

$a_y = -x^3(6\,xy) - 3(x^2y)(-3x^2)$

$= -6x^4y + 9x^4y = 3x^4y$

$$\vec{a}=3x^5\hat{i}+3x^4y\hat{j}$$

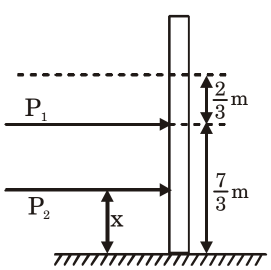
At (1, 1)

$$\vec{a}=3\hat{i}+3\hat{j}$$

$\Rightarrow |a| = \sqrt{3^2+3^2}$

$\Rightarrow |a| = 3\sqrt{2} = 4.24\ m/s^2$

18. According to the question;



$$P_1=\frac{(10^3\times 9.81\times 1)}{1000}$$

$P_1 = 9.81$ kPa

As we know that,

$$P_2=P_1+(\rho_2 g h_2)$$

$P_2 = P_1 + [2000 \times 9.81 \times 2]$

$P_2 = 49.05$ kPa

And, $$\bar{X}=\left[\frac{P_2+2P_1}{P_1+P_2}\right]\times\frac{2}{3}$$

By putting P_1 & P_2

$$\bar{X}=\frac{7}{9}$$

So, the resultant forces F_1 and F_2 due to pressure P_1 and P_2 respectively are :

Force (F_1) = Volume of triangular pressure prism

$$=\frac{1}{2}\times P_1\times 1\times 1=4.9\text{ kN}$$

Force (F_2) = Volume of trapezoidal pressure prism

$$=\frac{1}{2}\times[P_1+P_2]\times 2\times 1$$

$= 58.86$ kN

Now, the moment due to forces

$$M_o=F_1\times\left[2+\frac{1}{3}\right]+F_2\cdot\bar{X}$$

$$M_o=F_1\times\left[2+\frac{1}{3}\right]+F_2\times\frac{7}{9}$$

By putting the all values then, we get

$$M_o=\left[\left(4.9\times\frac{7}{3}\right)+\left(58.86\times\frac{7}{9}\right)\right]$$

$M_o = 57.225$ kNm

19. $\bar{v}=(x+2y+2)\hat{i}+(4-y)\hat{j}$

$u = x + 2y + 2,\ v = 4 - y$

$$\therefore \frac{\partial v}{\partial x}=1,\ \frac{\partial v}{\partial y}=-1$$

$$\therefore \frac{\partial v}{\partial x}+1,\frac{\partial v}{\partial y}=0,$$

hence incompressible.

Again, $$\omega=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial v}{\partial y}\right)$$

$$=\frac{1}{2}(0-2)=-1.$$

hence not irrotational.

20. Clearly zero shear stress and vortex.

21. The basic equation of continuity for fluid flow is given by

$$\frac{\rho u}{u}\quad\frac{\rho v}{y}\quad\frac{\rho w}{z}\quad\frac{\rho}{t}$$

Now if ρ remains constant, then only we can write

$$\vec{\nabla}\bullet\vec{V} = 0$$

i.e. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

hence incompressible flow

22. Incompressible flow condition

$$\nabla \cdot \vec{V} = 0$$

24. C is the false statement

2D incompressible flow continuity equation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial (Ax)}{\partial x} + \frac{\partial (-Ay)}{\partial y} = 0$$

A – A = 0 it satisfies continuity equation.

$\Rightarrow$ As $\vec{V} = Ax\,\hat{i} - Ay\hat{j}$

As $y \to \infty$ velocity vector field will not be defined along y axis.

So flow will be along x-axis i.e. 1-D flow.

$\Rightarrow$ Stream line equation for 2D

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{Ax} = \frac{dy}{-Ay}$$

$\ln x = -\ln y + \ln c$

$\ln xy = \ln c$

$xy = c \to$ streamline equation

25. x direction scalar of velocity field,

$$u = \frac{dx}{dt}$$

$u = -kx.e^{-kt}$

y direction scalar of velocity field

$$v = \frac{dy}{dt}$$

$v = ky_0e^{kt}$

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$\overline{V} = -kx_0e^{-kt}\hat{i} + ky_0e^{kt}\hat{j}$$

u & v are non zero scalar $t \geq 0$ so it is 2D flow.

2D possible flow field

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(-kx_0e^{-kt}\right) + \frac{\partial}{\partial y}\left(ky_0e^{kt}\right) = 0$$

0 + 0 = 0 continuity satisfied.

$$\frac{\partial u}{\partial t} = +k^2x_0e^{-kt}$$

$$\frac{\partial v}{\partial t} = k^2y_0e^{kt}$$

$$\frac{\partial u}{\partial t} \neq 0$$

$$\frac{\partial v}{\partial t} \neq 0$$

So, flow is unsteady.

26. As Given that

The velocity field of a two dimensional;

$$\vec{V} = 2\sin h\, x\hat{i} + v(x, y)\hat{j}$$

V(x, 0) = cos hx

For, incompressible flow $\nabla \cdot \vec{V} = 0$

$\Rightarrow \frac{\partial}{\partial x}(2\sin h\, x) + \frac{\partial v}{\partial y} = 0$

$\Rightarrow 2\cosh x + \frac{\partial v}{\partial y} = 0$

$\Rightarrow \frac{\partial v}{\partial y} = -2\cosh x$ ($\because$ By integrating both side)

$\Rightarrow$ v = –2y cosh x + f(x)

$\therefore$ v(x, 0) = cosh x $\Rightarrow$ f(x) = cosh x

$\therefore$ cosh x(1 – 2y)

$\Rightarrow$ v(0, –1) = (1(1 – 2(–1)) cos h(0) = 3

27. Given: $u_x = ax$ and $u_y = ay$

Equation of steam line is,

$$\frac{du}{u_x} = \frac{dy}{u_y} \Rightarrow \frac{du}{ax} = \frac{dy}{ay}$$

Integrating both sides, we have

$\log(ax) = \log(ay) + \log c$

or $ax = c \cdot ay$

or $x = cy$

Since the steam line is passed through point (1, 2), therefore

$1 = 2c$

$$\Rightarrow c = \frac{1}{2}$$

$$\therefore x = \frac{y}{2}$$

Hence equation of steam line is

$2x - y = 0$.

28. Given, $u = x^2t$ and $v = -2xyt$

We know, $\frac{\partial\psi}{\partial y} = v = -2xyt$ ...(*i*)

$\frac{\partial\psi}{\partial y} = -u = -x^2t$ ...(*ii*)

Integrating equation (*i*), we get

$\psi = -x^2yt + f(y)$ ...(*iii*)

Differentiating equation (*iii*) with respect to *y*, we get

$\frac{\partial\psi}{\partial y} = -x^2t + f(y)$ ...(*v*)

Equating the value of $\frac{\partial\psi}{\partial y}$ from equations (*ii*) and (*iv*), we get

$-x^2t = -x^2t + f'(y)$

Since, $f'(y) = 0$, thus $f(y) = C$

(where 'C' is constant of integration)

$\psi = -x^2yt + C$

C is a numerical constant so it can be taken as zero

$\psi = -x^2yt$

For equation of stream lines,

ψ = constant

$-x^2yt$ = constant

For a particular instance,

x^2y = constant

29. $\bar{v} = 2xyi - x^2zj$

Velocity of a vector = $\bar{\nabla} \times \bar{v}$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -x^2z & 0 \end{vmatrix}$$

$$= \hat{i}\left(0 - \frac{\partial}{\partial z}(-x^2z)\right) - \hat{j}\left(0 - \frac{\partial}{\partial z}(2xy)\right) + \hat{k}\left(\frac{\partial}{\partial x}(-x^2z) - \frac{\partial}{\partial y}(2xy)\right)$$

$= \hat{i}(-x^2) + \hat{k}(-2 \times z - 2x)$

At (1, 1, 1),

$\bar{\nabla} \times \bar{v} = \hat{i} + \hat{k}(-2-2) = i - 4k$

30. $\left(\frac{dy}{dx}\right)_\phi \times \left(\frac{dy}{dx}\right)_\Psi = -1$

Slope of equipotential Line

$\times$ slope of stream function $= -1$

They are orthogonal to each other.

33. Here $V(x, y) = u(x, y)\,\hat{i} + v(x, y)\,\hat{j}$

As we know that $u = \frac{-\partial\phi}{\partial x}$

$\therefore \frac{-\partial\phi}{\partial x} = -u(x, y)$ ...(*i*)

Similarly $v = \frac{-\partial\phi}{\partial y}$

$\therefore \frac{\partial\phi}{\partial y} = -v(x, y)$ ...(*ii*)

From equations (*i*) and (*ii*) we get,

$$\frac{\partial y}{\partial x} = \frac{\partial\phi}{\partial x} \times \frac{\partial y}{\partial\phi} = \frac{+u(x, y)}{-v(x, y)} = \frac{-u}{v}$$

34. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$a_1 + b_2 + c_3 = 0$

$2 - 4 + b_2 = 0$

$b_2 = 2$

36. $v = 2xy\,i - y^2j$

$u = \frac{\partial x}{\partial y}$, $v = -\frac{\partial x}{\partial x}$

$2xy = \frac{\partial x}{\partial y} = 2$

$2xy\partial y = \partial\psi$

on integrating

$\psi = xy^2 + f(x)$

Now $\frac{\partial x}{\partial x} = y^2 + f'(x) - v = y^2 + f'(x) - (-y^2)$

$= y^1 + f'(x)$

$f'(x) = 0$

$\Rightarrow$ f(x)= constant

so $\psi = xy^2$ + constant

37. $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$

$$= \frac{x}{x^2 + y^2}\left(\frac{x^2 + y^2 - x \times 2x}{(x^2 + y^2)^2}\right) - \frac{y}{(x^2 + y^2)} \times x \times \frac{1}{(x^2 + y^2)} \times 2y$$

$$= \frac{x(x^2 + y^2 - 2x^2) - 2xy^2}{(x^2 + y^2)(x^2 + y^2)} = \frac{-x^3 - xy^2}{(x^2 + y^2)}$$

$$\therefore \quad a_x = \frac{x}{(x^2+y^2)^2}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= \frac{x}{(x^2+y^2)} \times \frac{-y}{(x^2+y^2)^2} \times 2x \times \frac{y}{(x^2+y^2)} \times \left(\frac{(x^2+y^2) - y \times 2y}{(x^2+y^2)^2}\right)$$

$$= \frac{-2x^2y + yx^2 - y^3}{(x^2+y^2)^3} = \frac{-y}{(x^2+y^2)^2}$$

The velocity components are not functions of time, so flow is steady according to continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{-(x^2-y^2)}{(x^2+y^2)^2} + \frac{(x^2-y^2)}{(x^2+y^2)^2} = 0$$

Since it satisfies the above continuity equation for 2D and incompressible flow.

$\therefore$ The flow is incompressible.

38. $\vec{V} = (-x^2 + 3y)i + (2xy)j$

$u = -x^2 + 3y,\ v = 2xy$

$$\frac{\partial u}{\partial x} = -2x, \quad \frac{\partial v}{\partial x} = 2y$$

$$\frac{\partial u}{\partial y} = 3, \quad \frac{\partial v}{\partial y} = 2x$$

Now magnitude of particle at $(1, -1)$

$$u = -4,\ v = -2,\ \frac{\partial u}{\partial x} = -2,\ \frac{\partial u}{\partial y} = 3,$$

$$\frac{\partial v}{\partial x} = -2,\ \frac{\partial v}{\partial y} = 2$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (-4)\times(-2) + (-2)\times 3$$

$$a_x = 8 - 6 = 2$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = [(-4)\times(-2) + (-2)\times 2]$$

$$a_y = 8 - 4 = 4$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{2^2 + 4^2} = 2\sqrt{5}$$

39. From Navier stokes equation,

$\vec{V} = x\hat{i} - y\hat{j}$

$u = x,\ v = -y$

x-momentum

$$\cancelto{0}{\frac{\partial(\rho u)}{\partial t}} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \cancelto{0}{\frac{\partial(\rho uw)}{\partial z}}$$

$$= \frac{-\partial p}{\partial x} + \frac{1}{R_{cr}}\left[\cancelto{0}{\frac{\partial \tau_{xx}}{\partial x}} + \cancelto{0}{\frac{\partial \tau_{yy}}{\partial y}} + \cancelto{0}{\frac{\partial \tau_{zz}}{\partial t}}\right]$$

$$\frac{-\partial p}{\partial x} = \frac{\partial(\rho . x^2)}{\partial x} + \frac{\partial(\rho . x . -y)}{\partial y}$$

$$= 2\rho x - \rho x$$

$$\frac{\partial p}{\partial x} = -\rho x$$

Similarly, $\dfrac{\partial p}{\partial y} = -\rho y$

So, pressure gradient vector

$$= \frac{\partial p}{\partial y}\hat{i} + \frac{\partial p}{\partial y}\hat{j}$$

$$= -\rho x\hat{i} - \rho y\hat{j}$$

$$\boxed{\Delta p = -\rho(x\hat{i} + y\hat{j})}$$

4
CHAPTER

Fluid Dynamics

Bernoulli's Equation, Conservation of Energy and Mass

1. Navier Stoke's equation represents the conservation of

(a) Energy (b) Mass

(c) Pressure (d) Momentum

[2000 : 1 Mark]

2. The following data about the flow of liquid was observed in a continuous Chemical process plant:

Flow rate (litres / sec.)	7.5 to 7.7	7.7 to 7.9	7.9 to 8.1	8.1 to 8.3	8.3 to 8.5	8.5 to 8.7
Frequency	1	5	35	17	12	10

Mean flow rate of the liquid is

(a) 8.00 liters/s (b) 8.06 liters/s

(c) 8.16 liters/s (d) 8.26 liters/s

[2004 : 2 Marks]

3. Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from 20 cm to 10 cm. The pressure in the 20 cm pipe just upstream of the reducer is 150 kPa. The fluid has a vapour pressure of 50 kPa and a specific weight of 5 kN/m^3. Neglecting frictional effects, the maximum discharge (in m^3/s) that can pass through the reducer without causing cavitation is

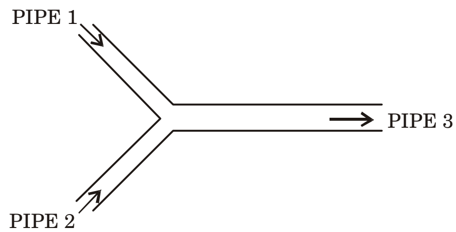
(a) 0.05 (b) 0.16

(c) 0.27 (d) 0.38

[2009 : 2 Marks]

4. Consider steady flow of water in a situation where two pipe lines (pipe 1 and pipe 2) combine into a single pipe line (pipe 3) as shown in figure. The cross-sectional'area of all three pipelines are constant. The following data is given

Pipe number	Area (m^2)	Velocity (m/s)
1	1	1
2	2	2
3	2.5	?



Assuming the water properties and the velocities to be uniform across the cross-section of the inlets and the outlet, the exit velocity (in m/s) in pipe 3 is

(a) 1 (b) 1.5

(c) 2 (d) 2.5

[2009 : 2 Marks]

5. Within a boundary layer for a steady incompressible flow, the Bernoulli equation

(a) holds because the flow is steady

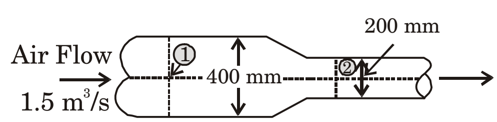
(b) holds because the flow is incompressible

(c) holds because the flow is transitional

(d) does not hold because the flow is frictional

[2015 : 1 Mark, Set-2]

6. Air flows at the rate of 1.5 m^3/s through a horizontal pipe with a gradually reducing cross-section as shown in the figure. The two cross-sections of the pipe have diameters of 400 mm and 200 mm. Take the air density as 1.2 kg/m^3 and assume inviscid incompressible flow. The change in pressure ($p_2 - p_1$) (in kPa) between sections 1 and 2 is

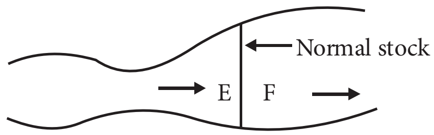


(a) –1.28 (b) 2.56

(c) –2.13 (d) 1.28

[2018 : 2 Marks, Set-2]

7. Steady compressible flow of air takes place through an adiabatic converging–diverging nozzle as shown in the figure. For a particular value of pressure differences across the nozzle, a stationary normal shock wave forms in the diverging section of the nozzle. If E & F denote the flow conditions just upstream & downstream of the normal shock respectively, which of the following statements is/are True?



(a) Density of E is lower than the density at F

(b) Static pressure at E is lower than its static processes at F

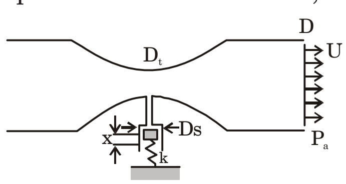
(c) Mach number at E is lower than the Mach number at F

(d) specific gravity at E is lower than specific gravity at F.

[2024 : 2 Marks]

Flow Measuring Devices

8. Air flows through a venturi and into atmosphere. Air density is ρ_a; atmospheric pressure is ρ_a; throat diameter is D_t; exit diameter is D and exit velocity is U. The throat is connected to a cylinder containing a frictionless piston attached to a spring. The spring constant is k. The bottom surface of the piston is exposed to atmosphere. Due to the flow, the piston moves by distance x. Assuming incompressible frictionless flow, x is



(a) $(\rho U^2/2k)\pi D_s^2$

(b) $(\rho U^2/8k)\left(\frac{D^2}{D_t^2}-1\right)\pi D_s^2$

(c) $(\rho U^2/2k)\left(\frac{D^2}{D_t^2}-1\right)\pi D_s^2$

(d) $(\rho U^2/8k)\left(\frac{D^4}{D_t^4}-1\right)\pi D_s^2$ **[2003 : 2 Marks]**

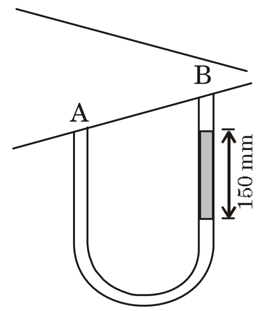
9. A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is

(a) 0.2 m/s (b) 1.0 m/s

(c) 1.4 m/s (d) 2.0 m/s

[2005 : 2 Marks]

10. A U-tube manometer with a small quantity of mercury is used to measure the static pressure difference between two locations A and B in a conical section through which an incompressible fluid flows. At a particular flow rate, the mercury column appears as shown in the figure. The density of mercury is 13600 kg/m^3 and g - 9.81 m/s^2. Which of the following is correct?



(a) Flow direction is A to B and $P_A - P_B = 20$ kPa

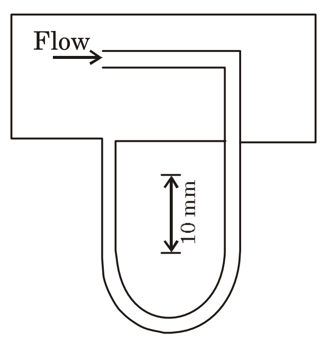
(b) Flow direction is Bto A and $p_A - p_B = 1.4$ kPa

(c) Flow direction is A to B and $p_B - p_A = 20$ kPa

(d) Flow direction is B to A and $p_B - p_A = 1.4$ kPa

[2005 : 2 Marks]

11. Figure shows the schematic for the measurement of velocity of air (density = 1.2 kg/m^3) through a constant-area duct using a pitot tube and a water tube manometer. The differential head of water (density = 1000 kg/m^3) in the two columns of the manometer is 10 mm. Take acceleration due to gravity as 9.8 m/s^2. The velocity of air in m/s is



(a) 6.4

(b) 9.0

(c) 12.8

(d) 25.6

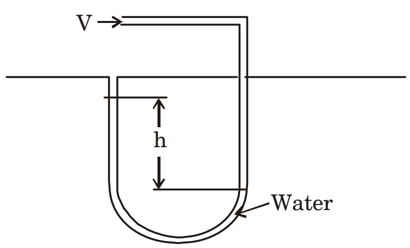
[2011 : 2 Marks]

12. A Prandtl tube (Pitot-static tube with C = 1) is used to measure the velocity of water. The differential manometer reading is 10 mm of liquid coiumn with a relative density of 10. Assuming g = 9.8 m/s², the velocity of water (in m/s) is _____ . **[2015 : 2 Marks, Set-3]**

13. Water (ρ = 1000 kg/m³) flows through a venturimeter with inlet diameter 80 mm and throat diameter 40 mm. The inlet and throat gauge pressures are measured to be 400 kPa and 130 kPa respectively. Assuming the venturimeter to be horizontal and neglecting friction, the inlet velocity (in m/s) is ________.

[2015 : 2 Marks, Set-1]

14. The arrangement shown in the figure measures the velocity V of a gas of density 1 kg/m³ flowing through a pipe. The acceleration due to gravity is 9.81 m/s². If the manometric fluid is water (density 1000 kg/m³) and the velocity V is 20 m/s, the differential head h(in mm) between the two arms of the manometer is _________.



[2017 : 2 Marks, Set-2]

15. A liquid fills a horizontal capillary tube whose one end is dipped in a large pool of the liquid. Experiment shows that the distance L travelled by the liquid meniscus inside the capillary in time t is given by

$$L = k.\gamma^a.R^b\mu^c.\sqrt{t}$$

Where γ is the surface tension. R is the inner radius of the capillary and μ is the dynamic viscosity of the liquid. If k is a dimensionless constant, the exponent a is ___ ***[Round off to the one decimal places]*** **[2024 : 2 Marks]**

Flow through Orifice and Mouthpiece

16. A water container is kept on a weighing balance. Water from a tap is falling vertically into the container with a volume flow rate of Q; the velocity of the water when it hits the water surface is U. At a particular instant of time the total mass of the container and water is m. The force registered by the weighing balance at this instant of time is

(a) mg + ρQU (b) mg + 2ρQU

(c) mg + ρQU²/2 (d) ρQU²/2

[2003 : 2 Marks]

17. Water is coming out from a tap and a falls vertically downwards. At the tap opening, the stream diameter is 20 mm with uniform velocity of 2 m/s. Acceleration due to gravity is 9.81 m/s². Assuming steady, inviscid flow, constant atmospheric pressure everywhere and neglecting curvature and surface tension effects, the diameter in mm of the stream 0.5 m below the tap is approximately

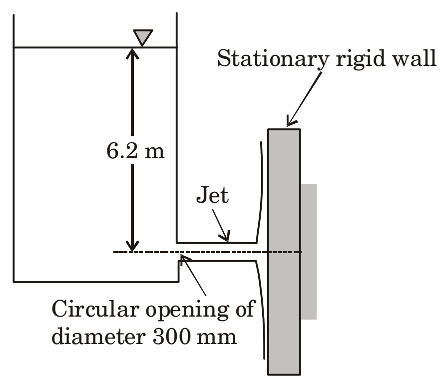
(a) 10 (b) 15

(c) 20 (d) 25

[2013 : 2 Marks]

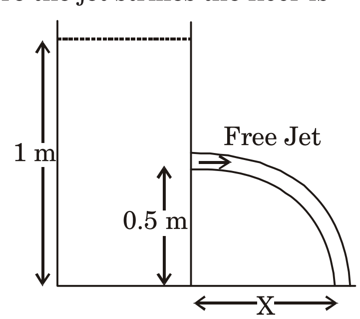
18. The water jet exiting from a stationary tank through a circular opening of diameter 300 mm impinges on a rigid wall as shown in the figure. Neglect all minor losses and assume the water level in the tank to remain constant. The net horizontal force experienced by the wall is ____ kN. Density of water is 1000 kg/m³.

Acceleration due to gravity g = 10 m/s².



[2016 : 2 Marks, Set-3]

19. A tank open at the top with a water level of 1 m, as shown in the figure, has a hole at a height of 0.5 m. A free jet leaves horizontally from the smooth hole. The distance X(in m) where the jet strikes the floor is



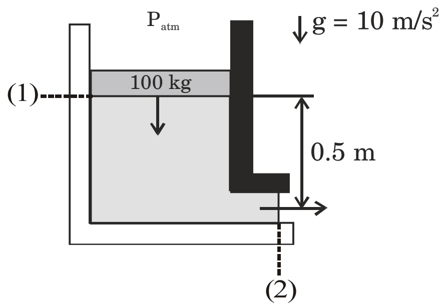
(a) 0.5 (b) 1.0

(c) 2.0 (d) 4.0

[2018 : 2 Marks, Set-1]

20. A frictionless circular piston of area $10^{-2}m^2$ and mass 100 kg sinks into a cylindrical container of the same area filled with water of density 1000 kg/m³ as shown in the figure.

The container has a hole of area $10^{-3}m^2$ at the bottom that is open to the atmosphere. Assuming there is no leakage from the edges of the piston and considering water to be incompressible, the magnitude of the piston velocity (in m/s) at the instant shown is _________ (correct to three decimal places).



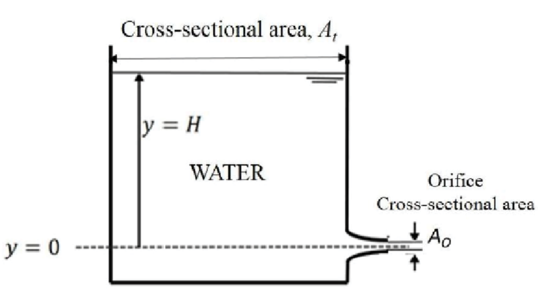
[2018 : 2 Marks, Set-2]

21. A pressure measurement device fitted on the surface of a submarine, located at a depth H below the surface of an ocean, reads an absolute pressure of 4.2 MPa. The density of sea water is 1050 kg/m³, the atmospheric pressure is 101 kPa, and the acceleration due to gravity is 9.8 m/s². The depth H is _______________ m

(*round off to the nearest integer*).

[2021 : 1 Mark, Set-1]

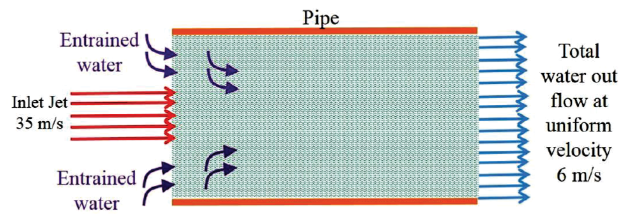
22. Water flows out from a large tank of cross-sectional area $A_t = 1\ m^2$ through a small rounded orifice of cross-sectional area $A_0 = 1\ cm^2$, located at y = 0. Initially the water level, measured from y = 0, is H = 1 m. The acceleration due to gravity is 9.8 m/s².



Neglecting any losses, the time taken by water in the tank to reach a level of y = H / 4 is ______seconds (*round off to one decimal place*).

[2021 : 2 Marks, Set-2]

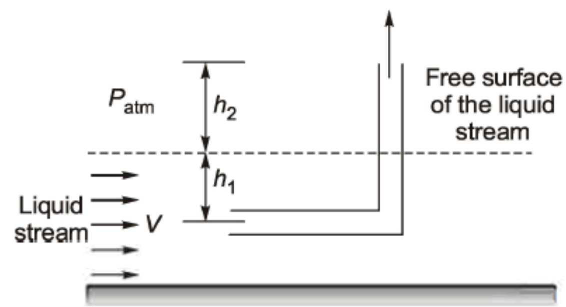
23. A high velocity water jet of cross section area = 0.01 m² and velocity = 35 m/s enters a pipe filled with stagnant water. The diameter of the pipe is 0.32 m. This high velocity water jet entrains additional water from the pipe and the total water leaves the pipe with a velocity 6m/s as shown in the figure.



The flow rate of entrained water is __________ litres/s (*round off to two decimal places*).

[2021 : 2 Marks, Set-2]

24. A tube of uniform diameter D is immersed in a steady flowing inviscid liquid stream of velocity V, as shown in the figure. Gravitational acceleration is represented by g. The volume flow rate through the tube is _______ .



(a) $\frac{\pi}{4}D^2V$

(b) $\frac{\pi}{4}D^2\sqrt{2gh_2}$

(c) $\frac{\pi}{4}D^2\sqrt{2g(h_1+h_2)}$

(d) $\frac{\pi}{4}D^2\sqrt{V^2-2gh_2}$

[2022 : 2 Marks, Set-2]

ANSWERS

1. (d) **2.** (c) **3.** (b) **4.** (c) **5.** (d) **6.** (a) **7.** (a,b,d) **8.** (d) **9.** (d) **10.** (a)
11. (c) **12.** (1.328) **13.** (6) **14.** (20.38) **15.** (0.5) **16.** (a) **17.** (b) **18.** (8.76) **19.** (b) **20.** (1.456)
21. 397 to 399 **22.** 2257.0 to 2259.0 **23.** 130.00 to 134.00 **24.** (d)

* Represents Numerical & Subjective Questions.

EXPLANATIONS

1. Momentum

2.

Flow rate (litre/sec)	Mean value of flow rate (x)	Frequency (f)	fx
7.5 – 7.7	7.6	1	7.6
7.7 – 7.9	7.8	5	39
7.9 – 8.1	8.0	35	280
8.1 – 8.3	8.2	17	139.4
8.3 – 8.5	8.4	12	100.8
8.5 – 8.7	8.6	10	86
		$\Sigma f = 80$	$\Sigma fx = 652.8$

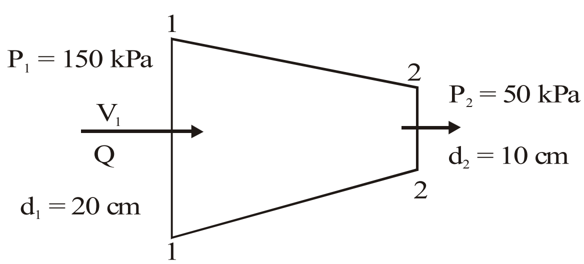
$\therefore$ Mean flow rate $= \dfrac{\Sigma fx}{\Sigma f} = \dfrac{652.8}{80} = 8.16.$

3. Considering potential head difference = 0,

i.e $z_1 = z_2$

Apply Bernoulli's theorem

$$\frac{p}{\rho g} + \frac{v^2}{2g} = C$$



$$\frac{p_1}{w_1} + \frac{v_1^2}{2g} = \frac{p_2}{w_2} + \frac{v_2^2}{2g}$$

But $w_1 = w_2 = w = 5$ (incompressible flow)

$$\therefore \quad \frac{150}{5} + \frac{v_1^2}{2g} = \frac{50}{5} + \frac{v_2^2}{2g}$$

or $$\frac{v_2^2 - v_1^2}{2g} = \frac{150-5}{5}$$

or $$\frac{v_2^2 - v_1^2}{2g} = 20\ m \qquad ....(1)$$

Also, discharge, $Q = \left(\dfrac{\pi}{4}d_1^2\right)v_1 = \left(\dfrac{\pi}{4}d_2^2\right)v_2$

or $$\frac{v_1}{v_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{10}{20}\right)^2$$

or $$v_1 = \frac{v_2}{4} \qquad ...(2)$$

From equations (1) and (2)

$$\frac{v_2^2 - \frac{v_2^2}{16}}{2g} = 20$$

or $$\frac{15v_2^2}{32g} = 20$$

or $$v_2 = \sqrt{\frac{20 \times 32g}{15}} = 20.45 \text{ m/s}$$

$\therefore$ Discharge, $Q = \left(\dfrac{\pi}{4}d_2^2\right)v_2$

$$= \frac{\pi}{4}(0.1)^2 \times 20.45 = 0.16 \text{ m}^3\text{/sec}$$

4. $Q1 + Q2 = Q3$

$A_1V_1 + A_2V_3 = A_3V_3$

$1 + 4 = 2.5\ V_3$

or $V_3 = 2$ m/s

5. Bernoulli equation does not hold because the flow is frictional.

6. $$\frac{p_1}{\delta g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\delta g} + \frac{{V_2}^2}{2g} + Z_2\,[Z = \text{constant}]$$

$$A_1 = \frac{\pi}{4} \times 0.4^2 = 0.1256 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\frac{P_2 - P_1}{\delta g} = \frac{V_1^2 - V_2^2}{2g}$$

$$\frac{P_2 - P_1}{1.2} = \frac{1}{2}\left[\frac{Q^2}{A_1^2} - \frac{Q^2}{A_2^2}\right]$$

$$P_2 - P_1 = 0.6 \times (1.5)^2\left[\frac{1}{A_1^2}\frac{-1}{A_2^2}\right]$$

$$P_2 - P_1 = 1.35\left[\frac{1}{0.01577} - \frac{1}{0.00985}\right]$$

$= 1.35\ [63.41 - 1015.22]$

$= -1.28 \times 10^3$ Pascal

$= -1.28$ kPa

7. a. $\rho_E < \rho_f$ Density after shock will increase $\rightarrow$ Correct statement.

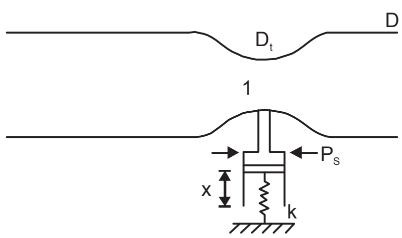
b. $\rho_E < \rho_f$ Static pressure at E is lower than the static process at F $\rightarrow$ correct statement.

c. $M_E < M_F \rightarrow$ wrong as velocity decreases across shock wave $\rightarrow$ wrong statement.

d. Entropy accross a shock wave always increases as it is highly irreversible process $\rightarrow$ correct statement .

Hence, options (a, b & d) are the correct answer.

8.



Applying Bernoulli's equation at points (1) and (2), we have

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Since venturi is horizontal

$z_1 = z_2$

$$\text{Now}\ \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\Rightarrow\ (P_1 - P_2) = \frac{\rho g}{2g}\left(v_2^2 - v_1^2\right) = \frac{\rho}{2}\left(v_2^2 - v_1^2\right)$$

Since $P_2 = P_a$ = atmospheric pressure

$$\therefore\ (P_1 - P_a) = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \qquad ...(i)$$

Applying continuity equation at points (*i*) and (*ii*), we have

$A_1 v_1 = A_2 v_2$

$$\Rightarrow\ v_1 = \left(\frac{A_2}{A_1}\right) v_2 \ \text{ since } V_2 = U$$

$$v_1 = \left(\frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} D_t^2}\right) U$$

$$\Rightarrow\ v_1 = \left(\frac{D}{D_t}\right)^2 U$$

From equation (*i*),

$$P_1 - P_a = \frac{\rho}{2}\left[v^2 - \left(\frac{D}{D_t}\right)^2 U^2\right]$$

$$= \frac{\rho}{2} U^2\left[1 - \frac{D^4}{D_t^4}\right]$$

At point P

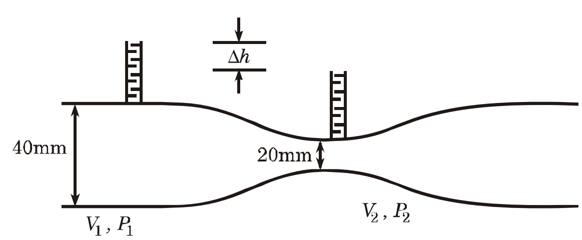
Spring force = pressure force due air

$$-kx = \frac{\pi}{4} D_s^2 \times \frac{\rho U^2}{2}\left[1 - \frac{D^4}{D_t^4}\right]$$

$$\Rightarrow x = \frac{\pi}{8}\frac{D_s^2 \rho U^2}{k}\left[1 - \frac{D^4}{D_t^4}\right]$$

9. We know, $A_1 V_1 = A_2 V_2$

$$Þ\quad V_2 = \frac{D_1^2}{D_2^2} V_1 = \frac{16}{4} V_1$$



$$\therefore\ \boxed{V_2 = 4V_1}$$

Applying Bernoulli's Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{eg} = \frac{V_2^2 - V_1^2}{2g}$$

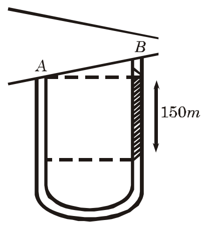
$$\Rightarrow \frac{15V_1^2}{2} = \frac{30 \times 10^3}{1000}$$

$$\Rightarrow V_1^2 = 4$$

$\Rightarrow V_1 = 2.0$ m/s

So velocity of flow is 2.0 m/sec

10.



$P_A - P_B = rg \cdot Dh$

$= 13600 \times 9.81 \times .15$

$= 20$ kPa

As pressure is decreasing from A to B, so flow direction is A to B.

11. From Bernoulli's equation

$$\frac{V_1^2 - V_2^2}{2g} = \frac{p_2 - p_1}{\rho_a g}$$

$$\Rightarrow V_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho_a}}$$

But $p_2 - p_1 = (\rho g h)_{water}$

$= 9810 \times 10 \times 10^{-3}$

$= 98.1$ N/m^2

$$\therefore V_1 = \sqrt{\frac{2 \times 98.1}{1.2}} = 12.8 \text{ m/s}$$

12. Velocity as water $= C_v\sqrt{2gh}$

$C_v = 1$ (Given)

$$h = x\left[\frac{s_g}{s_0} - 1\right]$$

$= 0.01\,(10 - 1) = 0.09$ m

$\therefore$ velocity of flow =

$\sqrt{2 \times 9.8 \times 0.09} = 1.328\,\text{m/s}$

13. $$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \begin{bmatrix} p_1 = 400000 \\ p_2 = 130000 \end{bmatrix}$$

$v_1 \times 80^2 = v_2 \times 40^2$

$v_2 = 4v_1$

Substituting v_2 and solving for v_1 we get

$v_1 = 6$m/s

14. Dynamic pressure of gas $= (\delta g h)_{water}$

$$\frac{1}{2}\delta_{gas} V_2 = \delta_m \times 9.81 \times h$$

$$\frac{1}{2} \times 1 \times (20)^2 = 1000 \times 9.81 \times h$$

h = 0.02038 m of water

h = 20.38 mm of water

15. As Given that

$$L = k.\gamma^a \cdot R^b \mu^c \cdot \sqrt{t}$$

$\Rightarrow [L] = [M^\circ L^\circ T^\circ][MT^{-2}]^a.[L]^b.[ML^{-1}T^{-1}]^c.[T]^{1/2}$

$$\Rightarrow [L] = \left[M^{a+c} L^{b-c} T^{-2a-c+\frac{1}{2}}\right]$$

$\Rightarrow a + c = 0 \Rightarrow c = -a$ ....(i)

$b - c = 1 \Rightarrow c = b - 1$ ....(ii)

$-2a - c + \frac{1}{2} = 0$ ....(iii)

By putting the value of c from eq (i) in equation (iii)

$$\Rightarrow -2a - (-a) + \frac{1}{2} = 0$$

$$\Rightarrow -a + \frac{1}{2} = 0$$

$\boxed{\Rightarrow a = 0.5}$

Now, from (i),

$c = -0.5$

and, $c = b - 1$

$b = c + 1$

$= -0.5 + 1$

$= 0.5$

Hence, the volume of a, b and c are 0.5, 0.5, –0.5.

16. Volume flow rate = Q

Mass of water strike = ρQ

Velocity of the water when it hit the water surface = U

Force on weighing balance due to water strike

= Initial momentum – final momentum

$= \rho QU - 0 = \rho QU$

(since final velocity is perpendicular to initial velocity)

Now total force on weighing balance

$= mg + \rho QU$

17. $Q = A_1 V_1 = A_2 V_2$

$$\Rightarrow \frac{\pi}{4}(20)^2 \times 2 = \frac{\pi}{4} \times d^2 \times \sqrt{2 \times 9.8 \times 0.5}$$

$d \approx 15$mm

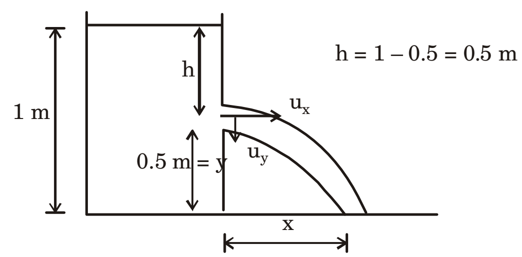
18. Force exerted by a jet of water striking fixed wall = $\rho a v^2$

$$= 1000 \times \frac{\pi}{4} \times 0.3^2 \times V^2$$

$$= 1000 \times \frac{\pi}{4} \times 0.3^2 \times \left(2 \times 10 \times 6^2\right) \left[\because\ V = \sqrt{2gh}\right]$$

= 8.76 N

19. $u_x = \sqrt{2gh}$



$$= \sqrt{2 \times 9.8 \times 0.5}$$

$u_x = 3.13$ m/s

Now for distance x

we require u_x so–

$$x = u_x \cdot t + \frac{1}{2} \times a \times b$$

$x = 3.13\,t$

($\because$ acceleration (a) = 0 in horizontal direction)

Now for $t \Rightarrow y = u_y t + \frac{1}{2} g t^2$

$$\Rightarrow 0.5 = 0 + \frac{1}{2} \times 9.81 \times t^2 \qquad [\because u_y = 0]$$

$t = 0.319$ sec.

so distance, $x = 3.13 \times 319 = 1$m

20. $A_1 V_1 = A_2 V_2$

$$V_2 = \left(\frac{A_2}{V_2}\right) V_1$$

By applying Bernoulli's equation between points (1) & (2), we get

$$\frac{P_1}{\delta g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\delta g} + \frac{V_2^2}{2g} + Z_2$$

$$\left[P_{atm} + \frac{100 \times 10}{10^{-2}}\right] + \frac{V_1^2}{2g} + 0.5 = \frac{p_{atm}}{\delta g} + \frac{A_1^2}{A_2^2} \times \frac{V_1^2}{2y}$$

$$\frac{100 \times 10}{10^{-2} \times 1000 \times 10} + 0.5 = \left(\frac{A_1^2}{A_2^2} - 1\right)\frac{V_1^2}{2g}$$

$$10 + 0.5 = \left(10^2 - 1\right)\frac{V_1^2}{2g} \Rightarrow V_1 = 1.456 \text{m/s}$$

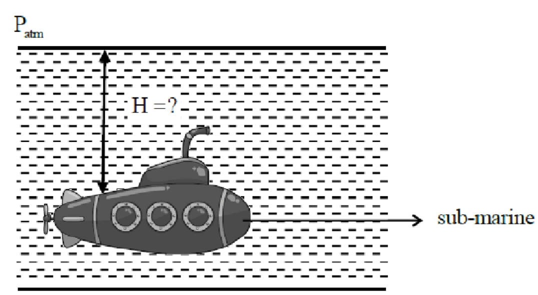
21. As given,

Density of sea water, $(\rho) = 1050$kg/m^3

$P_{atm} = 101$kPa

$= 101 \times 10^3$Pa $= 101 \times 10^3$ N/m^2

g = 9.8 m/sec^2



As we know that,

Pressure on the submarine,

$(P_{abs}) = P_{atm} + \rho g H$

$4.2 \times 10^6 = 101 \times 10^3 + (1050 \times 9.8 \times H)$

H = 398.35m

22. As given that,

Cross sectional area of tank $(A_t) = 1$m^2

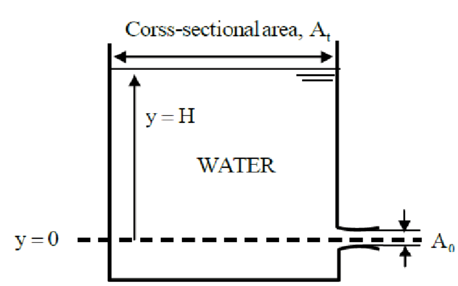
Cross sectional of area orifice $(A_0) = 1$cm^2

$= 1 \times 10^{-4}$ m^2

Initially Height of water, (H) = 1m

g = 9.8 m/sec^2

According to question.



Now, time taken to reach a level of water $y = \frac{H}{4}$ will be find out as follows,

$m_{in} - m_{out} = m_{cu}$

$m_{in} = 0$

$$-\rho A_0\sqrt{2gH} = \frac{d}{dt}\left(\rho A_t dH\right)$$

$$-A_0\sqrt{2gH} = \frac{d}{dt}\left(A_t dH\right)$$

$$-A_0\sqrt{2gH} = d\left(A_t dH\right)$$

By integration on both sides :

$$-A_0\sqrt{2gH}\left(t-0\right) = A_t\left(dH\right)$$

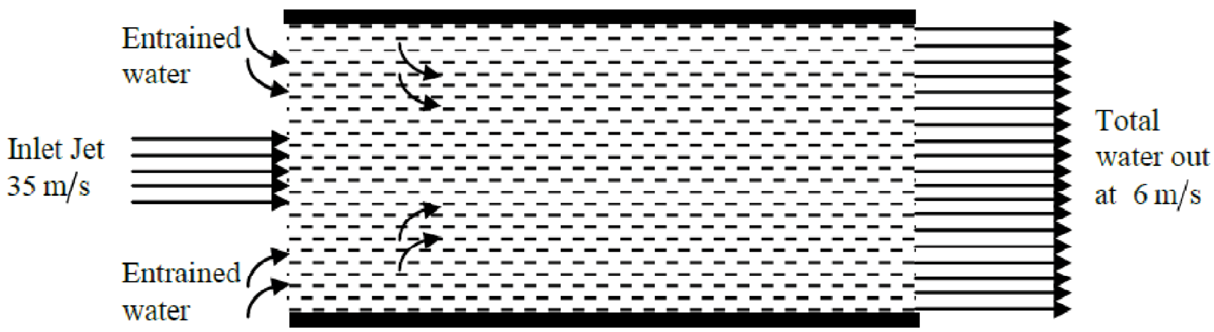
$$t = \frac{-A}{A_0\sqrt{2g}}\left(\frac{dH}{\sqrt{H}}\right)$$

$$t = \frac{-A_t}{A_0\sqrt{2g}}\int_1^{0.25}\frac{dH}{\sqrt{H}} = \frac{A}{A_0\left(\sqrt{29}\right)}\int_{0.25}^{1}\frac{dH}{\sqrt{H}}$$

$$= \frac{(1)}{\left(1\times10^{-4}\times\sqrt{2\times9.8}\right)}\left[2\sqrt{H}\right]_{0.25}^{1}$$

t = 2258 seconds

23. As given figure :



As given data,

d = diameter of jet, velocity of jet (V_{jet}) = 35 msec

a = cross-sectional area of jet = $0.01m^2$

D = Diameter of pipe = 0.32m,

V_0 = Total water outlet velocity = 6 m/sec

ρ = Density of water = 1000 kg/m^3

As we know that,

Cross-sectional area of pipe (A) = $\frac{\pi}{4}D^2$

$$A = \frac{\pi}{4}\left(0.32^2\right) = 0.080\,m^2$$

Now, applying conservation of mass of principle,

Mass flow rate of jet + Mass flow rate of entrained water (or) stagnant water = total mass flow rate ρaV_{jet} + e (flow rate stagnant water or entrained) = ρAV_0

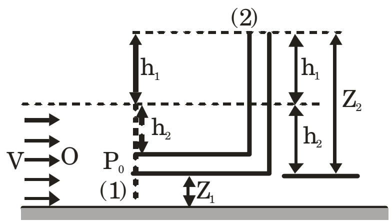
So, $aV_{jet} + (Q) = AV_0$

(0.01)(35) + (Q) = (0.080)(6)

Q = 0.13 m^3/sec = 130 litrers/sec

Hence, the flow rate of entrained water is 130 litrers/sec.

24. According to the question



Pressure at point (1) = $\rho g(h_1 + h_2)P_1$

At point 'O' where velocity of fluid is V is

$P_o = \rho g h_1$

Applying bernoulli's equation between point 1 and 2

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

So, $$\frac{P_1}{\rho g}+Z_1 = 0 + Z_2 \quad [\because A_1 = A_2 \Rightarrow V_1 = V_2]$$

$$P_1 = \rho g (Z_1 - Z_2)$$
$$= \rho g (h_1 + h_2)$$

$\therefore$ $$\boxed{P_0 = \rho g h_1}$$

$$\frac{P_0}{\rho g}+\frac{V^2}{2g}+Z_0 = \frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 \qquad (\therefore Z_0 = Z_1)$$

As $Z_0 = Z_1$

$$\frac{\rho g h_1}{\rho g}+\frac{V^2}{2g} = \frac{\rho g\left(h_1 + h_2\right)}{\rho g}+\frac{V_1^2}{2g}$$

$$\frac{V_1^2}{2g} = \frac{V^2}{2g} - h_2$$

$\Rightarrow$ $$V_1 = \sqrt{V^2 - 2gh_2}$$

So, $$Q = V_1 A_1 = \frac{\pi}{4}D^2\sqrt{V^2 - 2gh_2}$$

Hence, option (d) is the correct answer.

3 CHAPTER Second Law of Thermodynamics, Carnot Cycle and Entropy

Second Law of Thermodynamics

1. A cycle heat engine does 50 kJ of work per cycle. If the efficiency of the heat engine is 75%. The heat rejected per cycle is **[2001 : 2 Marks]**

(a) $16\frac{2}{3}$ kJ (b) $33\frac{1}{3}$ kJ

(d) $37\frac{1}{2}$ kJ (c) $66\frac{1}{2}$ kJ

2. A solar collector receiving solar radiation at the rate of 0.6 kW/m² transforms it to the internal energy of a fluid at an overall efficiency of 50%. The fluid heated to 350 K is used to run a heat engine which rejects heat at 313 K. If the heat engine is to deliver 2.5 kW power, the minimum area of the solar collector required would be

(a) 83.33 m² (b) 16.66 m²

(c) 39.68 m² (d) 79.36 m²

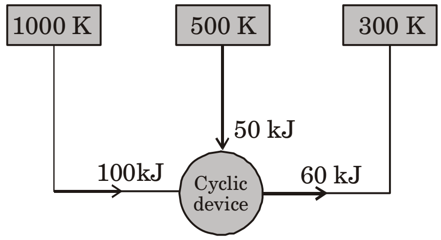
[2004 : 2 Marks]

3. A heat transformer is device that transfers a part of the heat, supplied to it at an intermediate temperature, to a high temperature reservoir while rejecting the remaining part to a low temperature heat sink. In such a heat transformer, 100 kJ of heat is supplied at 350 K. The maximum amount of heat in kJ that can be transferred to 400 K, when the rest is rejected to a heat sink at 300 K is

(a) 12.50 (b) 14.29

(c) 33.33 (d) 57.14 **[2007 : 2 Marks]**

4. A cyclic device operates between three thermal reservoirs, as shown in the figure. Heat is transferred to/from the cycle device. It is assumed that heat transfer between each thermal reservoir and the cyclic device takes place across negligible temperature difference. Interactions between the cyclic device and the respective thermal reservoirs that are shown in the figure are all in the form of heat transfer.



The cyclic device can be

(a) a reversible heat engine

(b) a reversible heat pump or a reversible refrigerator

(c) an irreversible heat engine

(d) an irreversible heat pump or an irreversible refrigerator.

[2008 : 2 Marks]

5. An irreversible heat engine extracts heat from a high temperature source at a rate of 100 kW and rejects heat to a sink at a rate of 50 kW. The entire work output of the heat engine is used to drive a reversible heat pump operating between a set of independent isothermal heat reservoirs at 17°C and 75°C. The rate (in kW) at which the heat pump delivers heat to its high temperature sink is

(a) 50 (b) 250

(c) 300 (d) 360

[2009 : 2 Marks]

6. Consider the following two processes;

I. A heat source at 1200 K loses 2500 kJ of heat to a sink at 800 K

II. A heat source at 800 K loses 2000 kJ of heat to a sink at 500 K

Which of the following statements is true?

(a) Process I is more irreversible than Process II

(b) Process II is more irreversible than Process I

(c) Irreversibility associated in both the processes are equal

(d) Both the processes are reversible

[2010 : 2 Marks]

7. A reversible heat engine receives 2 kJ of heat from a reservoir at 1000 K and a certain amount of heat from a reservoir at 800 K. It rejects 1 kJ of heat to a reservoir at 400 K. The net work output (in kJ) of the cycle is

(a) 0.8 (b) 1.0

(c) 1.4 (c) 2.0

[2014 : 2 Marks, Set-1]

8. A source at a temperature of 500 K provides 1000 kJ of heat. The temperature of environment is 27°C. The maximum useful work (in kJ) that can be obtained from the heat source is________. **[2014 : 1 Mark, Set-3]**

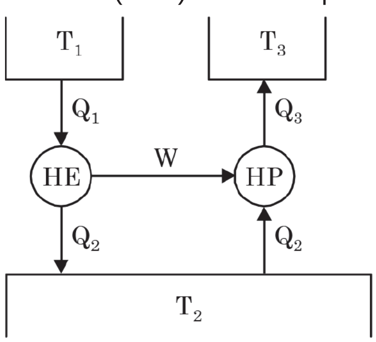
9. A Carnot engine (CE-1) works between two temperature reservoirs A and B, where T_A = 900 K and T_B = 500 K. A second Carnot engine (CE-2) works between temperature reservoirs B and C, where T_c = 300 K. In each cycle of CE-1 and CE-2, all the heat rejected by CE-1 to reservoir B is used by CE-2. For one cycle of operation, if the net Q absorbed by CE-1 from reservoir A is 150 MJ, the net heat rejected to reservoir C by CE-2 (in MJ) is ____. **[2015 : 1 Mark, Set-1]**

10. The heat removal rate from a refrigerated space and the power input to the compressor are 7.2 kW and 1.8 kW, respectively. The coefficient of performance (COP) of the refrigerator is____. **[2016 : 1 Mark, Set-2]**

11. A reversible cycle receives 40 kJ of heat from one heat source at a temperature of 127°C and 37 kJ from another heat source at 97°C. The heat rejected (in kJ) to the heat sink at 47°C is________ **[2016 : 2 Marks, Set-2]**

12. A heat pump absorbs 10 kW of heat from outside environment at 250K while absorbing 15 kW of work. It delivers the heat to a room that must be kept warm at 300 K. The Coefficient of Performance (COP) of the heat pump is____. **[2017 : 1 Mark, Set-1]**

13. The figure shows a heat engine (HE) working between two reservoirs. The amount of heat (Q_2) rejected by the heat engine is drawn by a heat pump (HP). The heat pump receives the entire work output (W) of the heat engine. If temperatures, $T_1 > T_3 > T_2$, then the relation between the efficiency (η) of the heat engine and the coefficient and the coefficient of performance (COP) of the heat pump is



(a) COP = 1 + η (b) COP = $\eta^{-1} - 1$
(c) COP = η^{-1} (d) COP = η
[2019 : 2 Marks, Set-2]

14. The Clausius inequality holds good for
(a) any process
(b) any cycle
(c) only reversible process
(d) only reversible cycle **[2022 : 1 Mark, Set-1]**

15. At steady state, 500 kg/s of steam enters a turbine with specific enthalpy equal to 3500 kJ/kg and specific entropy equal to 6.5 kJ·kg^{-1}·K^{-1}. It expands reversibly in the turbine to the condenser pressure. Heat loss occurs reversibly in the turbine at a temperature of 500 K. If the exit specific enthalpy and specific entropy are 2500 kJ/kg and 6.3 kJ·kg^{-1} ·K^{-1}, respectively, the work output from the turbine is __________ MW (italicize it). **[2022 : 2 Marks, Set-2]**

16. A heat engine extracts heat (Q_H) from a thermal reservoir at a temperature of 1000 K and rejects heat (Q_L) to a thermal reservoir at a temperature of 100 K, while producing work (W). Which one of the combinations of [Q_H, Q_L and W] given is allowed?
(a) Q_H = 2000 J, Q_L = 500 J, W = 1000 J
(b) Q_H = 2000 J, Q_L = 750 J, W = 1250 J
(c) Q_H = 6000 J, Q_L = 500 J, W = 5500 J
(d) Q_H = 6000 J, Q_L = 600 J, W = 5500 J
[2023 : 1 Mark]

Carnot Cycle

17. A Carnot cycle is having an efficiency of 0.75. If the temperature of the high temperature reservoir is 727°C. What is the temperature of low temperature reservoir?
(a) 23°C (b) –23°C
(c) 0°C (d) 250°C
[2001 : 2 Marks]

Entropy

18. If a closed system is undergoing an irreversible process, the entropy of the system
(a) must increase
(b) always remains constant
(c) must decrease
(d) can increase, decrease or remain constant
[2009 : 1 Mark]

19. One kilogram of water at room temperature is brought into contact with a high temperature thermal reservoir. The entropy change of the universe is
(a) equal to entropy change of the reservoir
(b) equal to entropy change of water
(c) equal to zero
(d) always positive **[2010 : 1 Mark]**

Common Data Question 20 and 21

In an experimental set-up, air flows between two stations P and Q adiabatically. The direction of flow depends on the pressure and temperature conditions maintained at P and Q. The conditions at station P are 150 kPa and 350 K. The temperature at station Q is 300 K. The following are the properties and relations pertaining to air:

Specific heat at constant pressure,

$c_p = 1.005$ kJ/kgK;

Specific heat at constant volume,

$c_v = 0.718$ kJ/kgK;

Characteristic gas constant,

R = 0.287 kJ/kgK.

Enthalpy, $h = c_pT$.

Internal energy, $u = c_vT$.

20. If the air has to flow from station P to station Q, the maximum possible value of pressure in kPa at station Q is close to

(a) 50 (b) 87

(c) 128 (d) 150 **[2011 : 2 Marks]**

21. If the pressure at station Q is 50 kPa, the change in entropy $(s_Q - s_p)$ in kJ/kgK is

(a) –0.155 (b) 0

(c) 0.160 (d) 0.355

[2011 : 2 Marks]

22. An ideal gas of mass m and temperature T_1 undergoes a reversible isothermal process from an initial pressure P_1 to final pressure P_2. The heat loss during the process is Q. The entropy change ΔS of the gas is

(a) $mR\ln\left(\frac{P_2}{P_1}\right)$

(b) $mR\ln\left(\frac{P_1}{P_2}\right)$

(c) $mR\ln\left(\frac{P_2}{P_1}\right) - \frac{Q}{T_1}$

(d) zero **[2012 : 1 Mark]**

23. Which one of the following pairs of equations describes an irreversible heat engine?

(a) $\oint \delta Q > 0$ and $\oint \frac{\delta Q}{T} < 0$

(b) $\oint \delta Q < 0$ and $\oint \frac{\delta Q}{T} < 0$

(c) $\oint \delta Q > 0$ and $\oint \frac{\delta Q}{T} > 0$

(d) $\oint \delta Q < 0$ and $\oint \frac{\delta Q}{T} > 0$

[2014 : 1 Mark, Set-3]

24. An amount of 100 kW of heat is transferred through a wall in steady State. One side of the wall is maintained at 127°C and the other side at 27°C. The entropy generated (in W/K) due to the heat transfer through the wall is ______.

[2014 : 2 Marks, Set-3]

25. A closed system contains 10kg of saturated liquid ammonia at 10°C. Heat addition required to convert the entire liquid into saturated vapour at a constant pressure is 16.2 MJ. If the entropy of the saturated liquid is 0.88 kJ/kgK, the entropy (in kJ/kgK) of saturated vapour is____

[2014 : 2 Marks, Set-4]

26. One kg of air (R = 287 J/kgK) undergoes an irreversible process between equilibrium state 1 (20°C, 0.9 m^3) and equilibrium state 2 (20°C, 0.6 m^3). The change in entropy $s_2 - s_1$ (in J/kgK) is ______. **[2015 : 2 Marks, Set-2]**

27. One kg of an ideal gas (gas constant, R= 400 J/kgK; specific heat at constant volume, $c_p = 1000$ J/kgK) at 1 bar, and 300 K is contained in a sealed rigid cylinder. During an adiabatic process, 100 kJ of work is done on the system by a stirrer. The increase in entropy of the system is _____ J/K. **[2017 : 2 Marks, Set-1]**

28. One kg of an ideal gas (gas constant R = 287 J/kgK) undergoes an irreversible process from state-1 (1 bar, 300 K) to state-2 (2 bar, 300 K). The change in specific entropy $(s_2 - s_1)$ of the gas (in J/kgK) in the process is _______.

[2017 : 2 Marks, Set-2]

29. An ideal gas undergoes a process from state 1 (T_1 = 300 K, p_1 = 100 kPa) to state 2 (T_2 = 600 K, p_2 = 500 kPa). The specific heats of the ideal gas are: c_p = 1 kJ/kg-K and c_v = 0.7 kJ/kg-K. The change in specific entropy of the ideal gas from state 1 to state 2 (in kJ/kg-K) is _____ (correct to two decimal places) **[2018 : 1 Mark, Set-1]**

30. For an ideal gas with constant properties undergoing a quasi-static process, which one of the following represents the change of entropy (Δs) from state 1 to 2?

(a) $\Delta s = c_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{R_1}\right)$

(b) $\Delta s = c_v\ln\left(\frac{T_2}{T_1}\right) - c_p\ln\left(\frac{V_2}{V_1}\right)$

(c) $\Delta s = c_p\ln\left(\frac{T_2}{T_1}\right) - c_p\ln\left(\frac{P_2}{P_1}\right)$

(d) $\Delta s = c_v\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_1}{V_2}\right)$

[2018 : 1 Mark, Set-2]

31. Air of mass 1 kg. initially at 300 K and 10 bar, is allowed to expand isothermally till it reaches a pressure of 1 bar. Assuming air as an ideal gas with gas constant of 0.287 kJ/kg K, the change in entropy of air (in kJ/kg K, round off to two decimal places) is ________.

[2019 : 1 Mark, Set-1]

32. One kg of air in a closed system undergoes an irreversible process from an initial state of $p_1 = 1$ bar (absolute) and $T_1 = 27°C$, to a final state of $p_2 = 3$ bar (absolute) and $T_2 = 127°C$. If the gas constant of air is 287 J/kg.K and the ratio of the specific heats g = 1.4, then the change in the specific entropy (in J/kg.K) of the air in the process is

(a) – 26.3

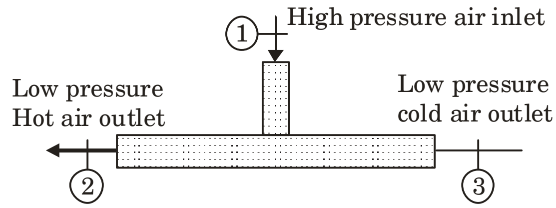
(b) 28.4

(c) 172.0

(d) indeterminate, as the process is irreversible

[2020 : 2 Marks, Set-2]

33. An adiabatic vortex tube, shown in the figure given below is supplied with 5 kg/s of air (inlet 1) at 500 kPa and 300 K. Two separate streams of air are leaving the device from outlets 2 and 3. Hot air leaves the device at a rate of 3 kg/s from outlet 2 at 100 kPa and 340 K, and 2 kg/s of cold air stream is leaving the device from outlet 3 at 100 kPa and 240 K.



Assume constant specific heat of air is 1005 J/kg.K and gas constant is 287 J/kg.K. There is no work transfer across the boundary of this device. The rate of entropy generation is _______ kW/K (*round off to one decimal place*).

[2021 : 2 Marks, Set-2]

ANSWERS

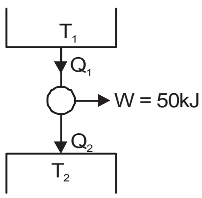
1. (a)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (400)	**9.** (50)	**10.** (4)
11. (64)	**12.** (1.67)	**13.** (c)	**14.** (b)	**15.** (450 to 450)		**16.** (b)	**17.** (b)	**18.** (a)	**19.** (d)
20. (b)	**21.** (c)	**22.** (b)	**23.** (a)	**24.** (83.33)		**25.** (6.6013)		**26.** (–116.36)	
27. (287.68)		**28.** (–198.93)		**29.** (0.21)	**30.** (a)	**31.** (0.66)	**32.** (a)	**33.** 2.1 to 2.3	

* Represents Numerical & Subjective Questions.

EXPLANATIONS

1. $\eta = 0.75$

$\therefore \ \eta_{HE} = \frac{W}{Q_1} = 0.75$



or, Net heat input,

$Q_1 = \frac{50}{0.75} = \frac{200}{3}$

Also, $W = Q_1 - Q_2$

or $50 = \frac{200}{3} - Q_2$

$\therefore$ Net heat required, $Q_2 = \frac{50}{3} = 16.66\,\text{kJ}$

2. Given; Receiving solar radiation at the rate of 0.6 kW/m^2

Internal energy of fluid after absorbing solar radiation

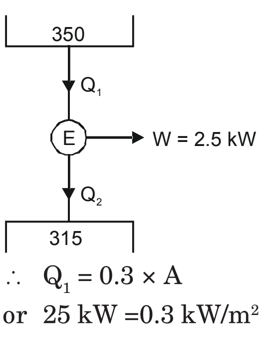
$= 0.6 \times \frac{1}{2} \text{kW/m}^2 = 0.3 \text{kW/m}^2$

$\eta_{engine} = 1 - \frac{315}{350} = 0.1$

$0.1 = \frac{W}{Q_1}$

$\therefore \ Q_1 = \frac{2.5}{0.1} = 25 \text{ kW}$

Let A be minimum area of collector



$\therefore \ Q_1 = 0.3 \times A$

or 25 kW =0.3 kW/m^2

or $A = \frac{25}{0.3} = 83.33 \text{ m}^2$

3. $\frac{Q_H}{Q_L} = \frac{400}{300} - \frac{4}{3}$

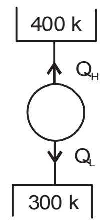
$Q_H + Q_L = Q = 100$

$\therefore \ \frac{4}{3}Q_L + Q_L = 100$

$\Rightarrow Q_L = 42.85 \text{ kJ}$

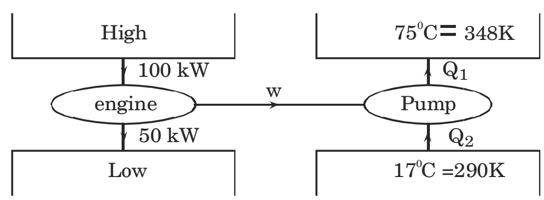
$\therefore \ Q_H = 100 - 42.85$

$= 57.14 \text{ kJ}$



4. Since heat is taken from the high temperature sources and rejected to low temperature sink, hence the device is a heat engine not heat pump. Since, the temperature differences are negligible, the engine is reversible.

5.



$W = Q_1 - Q_2$

$= 100 - 50$

$= 50 \text{ kW}$

$\text{C.O.P} = \frac{348}{348 - 290} = \frac{Q_1}{W}$

$Q_1 = 300 \text{ kW}$

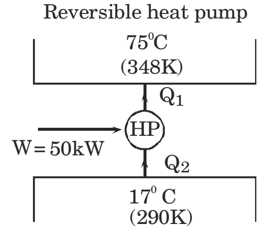
Alternately

Work output from irreversible heat engine,

$W = Q_1 - Q_2$

$= 100 - 50 = 50 \text{ kW}$

Reversible heat pump



Also for irreversible heat engine

$Q_1 + W = Q_2$

$\therefore T_1 \Delta S + 50 = T_2 \Delta S$

or $(T_2 - T_1)\,\Delta S = 50$

or $(348 - 290)\,\Delta S = 50$

or $\Delta S = \dfrac{50}{58}$

$\therefore$ Rate of heat rejection,

$Q_2 = T_2 \Delta S$

$= 348 \times \dfrac{50}{58} = 300$ kW

6. From Clausius inequality,

$$\oint \frac{\partial Q}{T} < 0 \text{ for irreversible process}$$

i.e., $\dfrac{Q_1}{T_1} + \dfrac{Q_2}{T_2} + \dfrac{Q_3}{T_3} + \ldots < 0$

For process I

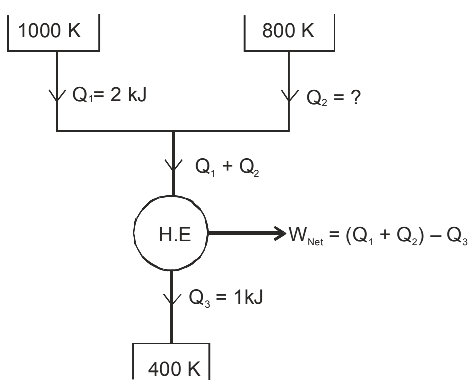
$$\oint \frac{\delta Q}{T} = \frac{2500}{1200} - \frac{2500}{800} = -1.042 < 0$$

For process II

$$\oint \frac{\delta Q}{T} = \frac{2000}{800} - \frac{2000}{500} = -1.5 < 0$$

Hence process II is more irreversible than process I.

7.



We know that for reversible heat engine, change in entropy is always zero

That is $\Delta S = 0$

$$\frac{Q_3}{T_3} - \left(\frac{Q_1}{T_1} + \frac{Q_2}{T_2}\right) = 0$$

$$\frac{1}{400} - \frac{2}{1000} - \frac{Q_2}{800} = 0$$

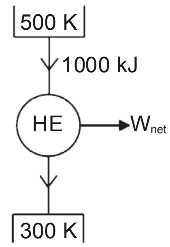
$Q_2 = 0.4$ kJ

$W_{Net} = (Q_1 + Q_2) - Q_3 = (2 + 0.4) - 1 = 1.4$ kJ.

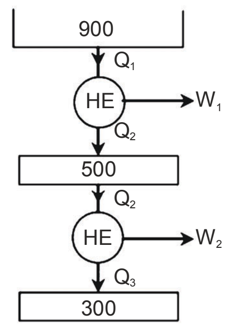
8. $\eta = \dfrac{W_{net}}{Q_{in}} = 1 - \dfrac{T_{sink}}{T_{source}}$

$$1 - \frac{300}{500} = \frac{W_{net}}{1000}$$

$W_{net} = 1000 \times 0.4 = 400$ kJ.



9.



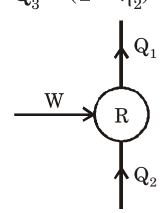
$$\eta_1 = \frac{1 - T_1}{T_1} = \frac{1 - 500}{900} = 4.44$$

$Q_2 = (1 - \eta_1) \times Q_1 = 53.33$ MJ

$$\eta_2 = 1 - \frac{T_3}{T_2} = 1 - \frac{300}{500} = 0.4$$

$Q_3 = (1 - \eta_2) \times Q_2 = 50$ MJ

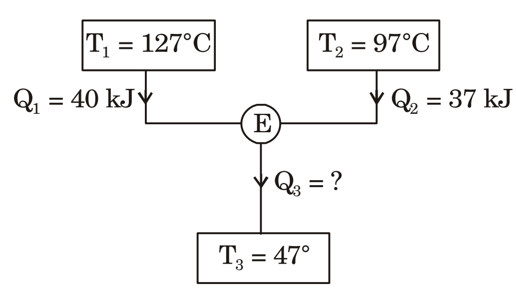
10.



$$\text{COP} = \frac{Q_2}{W} = \frac{7.2}{1.8}$$

COP = 4

11.

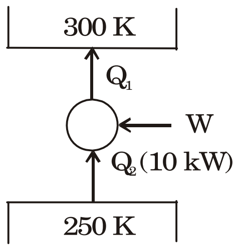


$$\frac{Q_1}{T_1}+\frac{Q_2}{T_2}-\frac{Q_3}{T_3}>0$$

$$\frac{40}{127+273}+\frac{37}{97+273}-\frac{Q_3}{47+273}=0$$

$$Q_3 = 64 \text{ kJ}$$

12.



By question, given:

$Q_2 = 10\text{kW}$; $W = 15\text{kW}$

$Q_1 = Q_2 + W = 25\text{kW}$

$$\text{COP} = \frac{Q_1}{W} = 1.67$$

13. Efficiency of heat engine,

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$(\text{COP})_{\text{H.P.}} = \frac{Q_3}{W}$$

$\because \quad Q_3 = Q_2 + W = Q_2 + Q_1 - Q_2$

$\quad Q_3 = Q_1$

so, $(\text{COP})_{\text{H.P}} = \dfrac{Q_1}{W}$

$$\boxed{(\text{COP})_{\text{H.P}} = \frac{1}{\eta} = \eta^{-1}}$$

14. As we know that

The Clausius inequality holds good for any cycle.

$$\oint \frac{dQ}{T} \le 0$$

$$\oint \frac{dQ}{T} = 0 \Rightarrow \text{Valid for Reversible cycle}$$

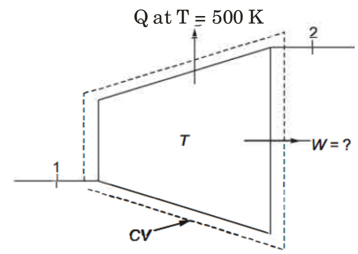
$$\oint \frac{dQ}{T} < 0 \Rightarrow \text{Valid for Irreversible cycle}$$

$$\oint \frac{dQ}{T} > 0 \Rightarrow \text{Impossible cycle}$$

Hence, the inequality is valid for any cycle.

15. According to the Question;

$$\dot{S}_{gen} = 0$$



As given that

$m_S = 500$ kg/sec

$S_1 = 6.5$ kJ/kgk, $S_2 = 6.3$ kJ/kgK

$h_1 = 350$ kJ/kg, $h_2 = 2500$ kJ/kg

Entropy generated can be found by using following relations

$$S_2 - S_1 = \sum \frac{Q_j}{T_j} + S_{gen}$$

For reversible process

$$\dot{S}_{gen} = 0$$

$$\underbrace{\cancel{\left(\frac{dS}{dt}\right)_{CV}}}_{0\text{(Steady state)}} = \dot{S}_i + \overbrace{\cancel{\dot{S}_{gen}}}^{0\text{(Reversible)}} - \dot{S}_e$$

$$0 = \dot{m}s_1 - \left\{\dot{m}s_2 + \frac{Q}{500}\right\}$$

$$0 = 500 \times \{6.5 - 6.3\} - \frac{Q}{500}$$

$$\dot{Q} = 50000 \text{ kW}$$

$$\dot{Q} = 50 \text{ MW}$$

Now, by conservation of energy :

$$\dot{E}_{in} = \dot{E}_{exit}$$

$$\dot{m}h_1 = \dot{m}h_2 + \dot{Q} + \dot{W}_T$$

$$\dot{m}(h_1 - h_2) = \dot{Q} + \dot{W}_T$$

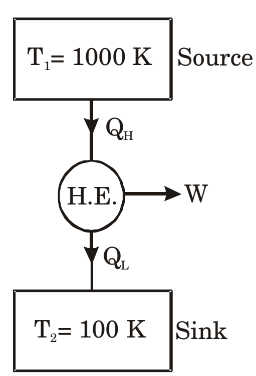
$$500\,(3500 - 2500) = 50000 + \dot{W}$$

$$\dot{W}_T = 450000 \text{ kW}$$

$$\dot{W}_T = 450 \text{ MW}$$

16. As we know that,

The rate of heat rejection is minimum for a reversible engine.



So, for feasible process :

$$\oint \frac{\delta Q}{T} \le 0$$

By taking option (b);

$$\oint \frac{\delta Q}{T} = \left(\frac{Q_H}{T_1}\right) - \left(\frac{Q_L}{T_2}\right)$$

$$= \frac{2000}{1000} - \frac{750}{100} = 2 - 7.5$$

$$= -5.5 < 0$$

Hence, option (b) is correct answer.

17. $1 - \dfrac{T_2}{T_1} = 0.75$

$\dfrac{T_2}{273 + 727} = 0.25$

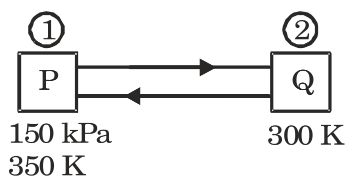
$T_2 = 250K$

$T_2 = 273 - 250$

$[T_2 - 23°C]$

18. $(ds)_{uni} \ge 0$

19.



$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\Rightarrow \quad P_2 = \left(\frac{300}{350}\right)^{\frac{0.4}{1.4}} \times 150$$

$P_2 = 87.4$ kPa

$P_2 = 87$kPa

20. $S_Q - S_P = C_v \ln\left(\dfrac{T_2}{T_1}\right) + R\ln\left(\dfrac{v_2}{v_1}\right)$

From perfect gas law,

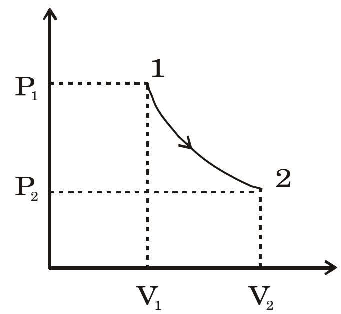
$$\frac{v_2}{v_1} = \frac{P_1}{P_2}\frac{T_2}{T_1}$$

$$= \frac{150 \times 300}{300 \times 350} = 2.57$$

$\therefore S_Q - S_p = -0.1107 + 0.287 \ln 2.57$

$= 0.16$ kJ/kgK

21. $\Delta s = \dfrac{\delta q}{T}$ for reversible process



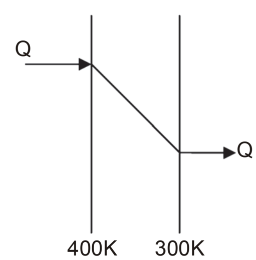
$$\delta Q = \delta W = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$\Delta s = \frac{\delta q}{T} = mR\ln \frac{p_1}{p_2}$$

$$\left[\Delta s = mR\ln \frac{p_1}{p_2}\right].$$

22. For clausius theorem, $\oint \dfrac{\delta Q}{T} < 0$ (for irreversible Heat engine).

24. $\Delta S_1 = \dfrac{Q}{T_1}$



$\Delta S_2 = \dfrac{Q}{T_2}$

$(\Delta S)_{generated} = \Delta S_1 + \Delta S_2$

$= \dfrac{Q}{400} - \dfrac{Q}{300}$

$= \dfrac{100 \times 10^3}{100}\left(\dfrac{1}{4} - \dfrac{1}{3}\right) = 10^3 \times -0.0833$

$= -83.33$ W / K

Entropy generated = 83.33 W/K.

25. Tds = du + pdv (closed system)

$283.15 \times 10(S_2 - S_1) = 16.2 \times 10^3$

$\therefore \quad S_2 = 0.88 + \dfrac{16.2 \times 10^3}{283.15 \times 10}$

= 6.6013 kJ/kg

26. –116.36 J/kg–K

$\Delta S = mR\ln \dfrac{V_2}{V_1}$

$= (287)(1)\ln\left(\dfrac{0.6}{0.9}\right)$

$\Rightarrow -116.36 \text{ J/kgK}$

27. Here,

$m = 1$ kg

R = 400 J/kgK

$C_V = 1000$ J/kgK

$P_1 = 1$ bar

$T_1 = 300$ K

$W = -100 \text{ kJ} = -100 \times 10^3$ J

Q = 0

$0 = -100 \times 10^3 + mC_V(T_2 - T_1)$

$100 \times 10^3 = 1 \times 1000\,(T_2 - 300)$

$T_2 = 400$ K

$V_1 = V_2$

$\Rightarrow \quad \ln \dfrac{V_2}{V_1} = 0$

$S_2 - S_1 = mC_V \ln \dfrac{T_2}{T_1}$

$S_2 - S_1 = 1 \times 1000 \times \ln\left(\dfrac{400}{300}\right)$

$S_2 - S_1 = 287.68$ J/K

28. $\Delta S = mC_p \ln \dfrac{T_2}{T_1} - R\ln \dfrac{P_2}{P_1}$

$\Delta S = -287\ln 2$

$\Delta S = -198.93$ J/kg K

29. Ideal gas

State - 1 : $T_1 = 300$ K, $P_1 = 100$ kPa

State - 2 : $T_2 = 600$ K, $P_2 = 500$ kPa,

$c_p = 1$ kJ/kg-K, $c_p - c_v = R$, $c_v = 0.7$ kJ/kg-K

$\Rightarrow \quad c_p - c_v = 1 - 0.7 = R$

R = 0.3 kJ/kg–K

Change in specific entropy

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= 1 \times \ln\frac{600}{300} - 0.3\ln\frac{500}{100} = 0.21 \text{ kJ/kg-K}$$

30. $\Delta S = C_p \ln \dfrac{T_2}{T_1} - R\ln \dfrac{P_2}{P_1}$

Tds = dh – vdp

$= C_p dt - vdp$

$ds = \dfrac{C_p dt}{T} - \dfrac{R}{p}dp$

$\left[ds = C_P \ln \dfrac{T_2}{T_1} - R\ln\dfrac{P_2}{P_1}\right]$

31. Air, $m = 1$ kg

$T = 300\text{K} \xrightarrow{\text{Isothermally}} T = 300\text{K}$

$P_2 = 1$ bar

$P_1 = 10$ bar

R = 0.287 kJ/kg k

Now, $\Delta s = S_2 - S_1$

$= mc_p \ell n\left(\dfrac{T_2}{T_1}\right)^{O} - mR\ell n\left(\dfrac{P_2}{P_1}\right)$

$= -1 \times 0.287 \ln\left(\dfrac{1}{10}\right)$

= 0.66 kJ/kg k

$\therefore \quad \Delta s = 0.66$ kJ/kg k

32. Closed system undergoing irreversible process,

Air, m = 1 kg, R = 0.287 kJ/kgk, $\gamma = 1.4$

Initial state	Final state
$P_1 = 1$ bar (absolute)	$P_2 = 3$ bar (absolute)
$T_1 = 27°C = 300K$	$T_2 = 127°C = 400$ K

Change in the specific entropy of the air in the process,

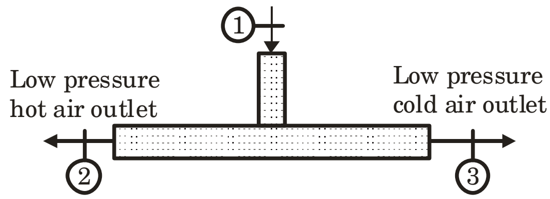
$$\Delta S = S_2 - S_1$$

$$= C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.005\ln\left(\frac{400}{300}\right) - 0.287\ln\left(\frac{3}{1}\right)$$

$$= -0.0262\frac{\text{kJ}}{\text{kgk}} = -26.2\frac{\text{J}}{\text{kgk}}$$

33. As given figure



As given that

m_1 = 5 kg/sec, P_1 = 500 kPa, T_1 = 300K

m_2 = 3kg/sec, P_2 = 100 kPa, T_2 = 340K

m_3 = 2kg/sec, P_3 = 100 kPa, T_3 = 240K

C_p =1005 J/kgK; gas constant (R) = 287J / kgK

$R = c_p - c_v \Rightarrow c_v = c_p - R = 1005 - 287 = 718$ J/kgK

Since tube is maintained in adiabatic condition

No entropy transfer from the vortex tube

$\therefore$ Entropy generation = Entropy change – Entropy transfer

$\therefore$ Entropy generation = Entropy change ($\because$ entropy transfer = 0)

As we know that

Entropy change of hot air

$$= m_2\left[c_p \ell n\left(\frac{T_2}{T_1}\right) - R\ell n\left(\frac{P_2}{P_1}\right)\right]$$

$$= (3)\left[(1005)\ell n\left(\frac{340}{300}\right) - (287)\ell n\left(\frac{100}{500}\right)\right]$$

= 1763.1 W/K

Entropy change of cold air

$$= m_3\left[c_p \ell n\left(\frac{T_3}{T_1}\right) - R\ell n\left(\frac{P_3}{P_1}\right)\right]$$

$$= (2)\left[(1005)\ell n\left(\frac{240}{300}\right) - (287)\ell n\left(\frac{100}{500}\right)\right]$$

= 475.30 W/K

$\therefore$ Entropy generation (s_{gen})

= Total entropy change

= 1763.1 + 475.30

= 2238.4 W/K = 2.24 kW/K