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Graduate Aptitude Test in Engineering

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Solved Papers

Electronics & Communication Engineering

G. K. PUBLICATIONS (P) LTD.

Title : **GATE 2025 :** Electronics & Communication Engineering - 25 Years' Topic-wise Previous Solved Papers

Language : English

Editor's Name : GKP Editorial Team

Typeset & Published by :

Career Launcher Infrastructure (P) Ltd.

A-45, Mohan Cooperative Industrial Area, Near Mohan Estate Metro Station, New Delhi - 110044

Marketed by :

G.K. Publications (P) Ltd.

Plot No. 63, Sector-27A, Near Sector - 28 Metro Station, Faridabad, Haryana-121003

ISBN : **978-93-5681-975-7**

Printer's Details : Printed in India, New Delhi.

For product information :

Visit ***www.gkpublications.com*** or email to ***gkp@gkpublications.com***

GATE
Electronics & Communication Engineering

Contents

Solved Papers (Topic-Wise)

6 CHAPTER

Z-transform

Z-TRANSFORM OF DISCRETE SIGNALS

1. The region of convergence of the z-transform of a unit step function is

(a) $|z| > 1$

(b) $|z| < 1$

(c) (Real part of z) > 0

(d) (Real part of z) < 0

[2001 : 1 Mark]

2. If the impulse response of a discrete-time system is $h[n] = -5^n u[-n-1]$, then the system function H(z) is equal to

(a) $\frac{-z}{z-5}$ and the system is stable

(b) $\frac{z}{z-5}$ and the system is stable

(c) $\frac{-z}{z-5}$ and the sVstem is unstable

(d) $\frac{z}{z-5}$ and the system is unstable

[2002 : 2 Marks]

3. A sequence x(n) with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The output at n = 4 is

(a) –6 (b) zero

(c) 2 (d) –4

[2003 : 1 Mark]

4. The z-transform of a system is $H(z) = \frac{z}{z-0.2}$.

If the ROC is $|z| < 0.2$, then the impulse response of the system is

(a) $(0.2)^n u[n]$

(b) $(0.2)^n u[-n-1]$

(c) $-(0.2)^n u[n]$

(d) $-(0.2)^n u[-n-1]$

[2004 : 1 Mark]

5. A causal LTI system is described by the difference equation

$2y[n] = \alpha y[n-2] - 2x|n| - \beta x[n-1]$.

The system is stable only if

(a) $|\alpha| = 2$, $|\beta| < 2$

(b) $|\alpha| > 2$, $|\pi| > 2$

(c) $|\alpha| < 2$, any value of β

(d) $|\beta| < 2$, any value of α

[2004 : 2 Marks]

6. The region of convergence of z-transform of the sequence

$\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be

(a) $|z| < \frac{5}{6}$ (b) $|z| > \frac{5}{6}$

(c) $\frac{5}{6} < |z| < \frac{6}{5}$ (d) $\frac{6}{5} < |z| < \infty$

[2005 : 1 Mark]

7. If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$, then the region of convergence of $x_1[n] - x_2[n]$ includes

(a) $\frac{1}{3} < |z| < 3$ (b) $\frac{2}{3} < |z| < 3$

(c) $\frac{3}{2} < |z| < 3$ (d) $\frac{1}{3} < |z| < \frac{2}{3}$

[2006 : 1 Mark]

8. The z-transform X[z] of a sequence x[n] is given by $X[z]=\frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of X[z] includes the unit circle. The value of x[0] is

(a) –0.5 (b) 0
(c) 0.25 (d) 0.5

[2007 : 2 Marks]

9. The ROC of z-transform of the discrete time sequence

$$x(n)=\left(\frac{1}{3}\right)^n u(n)-\left(\frac{1}{2}\right)^n u(-n-1) \text{ is}$$

(a) $\frac{1}{3}<|z|<\frac{1}{2}$ (b) $|z|>\frac{1}{2}$
(c) $|z|<\frac{1}{3}$ (d) $2<|z|<3$

[2009 : 1 Mark]

10. Consider the z-transform $X(z) = 5z^2 + 4z^{-1} + 3$; $0 < |z| < \infty$. The inverse z-transform x[n] is

(a) $5\delta[n + 2] + 3\delta[n] + 4\delta[n - 1]$.
(b) $5[n - 2] + 3\delta[n] + 4\delta[n + 1]$.
(c) $5u[n + 2] + 3u[n] + 4u[n - 1]$.
(d) $5u[n - 2] + 3u[n] + 4u[n + 1]$.

[2010 : 1 Mark]

11. The transfer function of a discrete time LTI system is given by

$$H(z)=\frac{2-\frac{3}{4}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

Consider the following statements:

S_1 : The system is stable and causal for ROC : $|z|>\frac{1}{2}$

S_2 : The system is stable but not causal for ROC : $|z|<\frac{1}{4}$

S_3 : The system is neither stable nor causal for ROC : $\frac{1}{4}<|z|<\frac{1}{2}$

Which one of the following statements is valid?

(a) Both S_1 and S_2 are true
(b) Both S_2 and S_3 true
(c) Both S_1 and S_3 are true
(d) S_1, S_2 and S_3 are all true

[2010 : 2 Marks]

12. If $x[n]=\left(\frac{1}{3}\right)^{|n|}-\left(\frac{1}{2}\right)^n u[n]$, then the region of convergence (ROC) of its z-transform in the z-plane will be

(a) $\frac{1}{3}<|z|<3$ (b) $\frac{1}{3}<|z|<\frac{1}{2}$
(c) $\frac{1}{2}<|z|<3$ (d) $\frac{1}{3}<|z|$

[2012 : 1 Mark]

13. C is a closed path in the z-plane given by $|z| = 3$. The value of the integral $\oint_c\left(\frac{z^2-z+4j}{z+2j}\right)dz$ is

(a) $-4\pi(1 + j2)$ (b) $4\pi(3 - j2)$
(c) $-4\pi(3 + j2)$ (d) $4\pi(1 - j2)$

[2014 : 1 Mark, Set-1]

14. Let $x[n]=\left(-\frac{1}{9}\right)^n u(n)-\left(-\frac{1}{3}\right)^n u(-n-1)$.

The Region of Convergence (ROC) of the z-transform of x[n]

(a) is $|z|>\frac{1}{9}$ (b) is $|z|<\frac{1}{3}$
(c) is $\frac{1}{3}>|z|>\frac{1}{9}$ (d) does not exist

[2014 : 2 Marks, Set-1]

15. Let x[n] = x[–n]. Let X(z) be the z-transform of x[n]. If 0.5 + j0.25 is a zero of X(z), which one of the following must also be a zero of X(z).

(a) 0.5 – j0.25 (b) $\frac{1}{(0.5+j0.25)}$
(c) $\frac{1}{(0.5-j0.25)}$ (d) 2 + j4

[2014 : 1 Mark, Set-2]

16. The input-output relationship of a causal stable LTI system is given as

$y[n] = \alpha y[n - 1] + \beta x[n]$. If the impulse response h[n] of this system satisfies the condition $\sum_{n=0}^{\infty}h[n]=2$, the relationship between α and β is

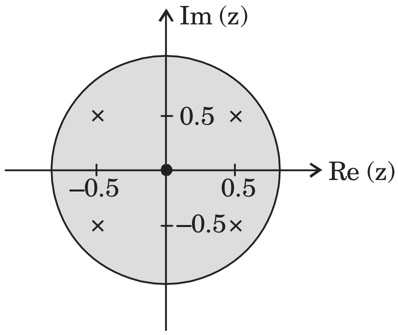
(a) $\alpha=1-\frac{\beta}{2}$ (b) $\alpha=1+\frac{\beta}{2}$
(c) $\alpha = 2\beta$ (d) $\alpha = -2\beta$

[2014 : 2 Marks, Set-2]

17. The z-transform of the sequence x[n] is given by $X(z)=\dfrac{1}{(1-2z^{-1})^2}$, with the region of convergence $|z|>2$. Then x[2] is ______.

[2014 : 2 Marks, Set-3]

18. The pole-zero diagram of a causal and stable discrete-time system is shown in the figure. The zero at the origin has multiplicity 4. The impulse response of the system is h[n]. If h[0] = 1, we can conclude



(a) h[n] is real for all n

(b) h[n] is purely imaginary for all n

(c) h[n] is real for only even n

(d) h[n] is purely imaginary for only odd n

[2015 : 2 Marks, Set-1]

19. Two causal discrete-time signalsx[n] and y[n] are related as $y[n]=\sum_{m=0}^{n}x[m]$. If the z-transform of y[n] is $\dfrac{2}{z(z-1)^2}$, the value of x[2] is _______. **[2015 : 1 Mark, Set-2]**

20. Suppose x[n] is an absolutely summable discrete- time signal. Its z-transform is a rational function with two poles and two zeroes. The poles are at $z=\pm 2j$. Which one of the following statements is TRUE for the signal x[n]?

(a) It is a finite duration signal.

(b) It is a causal signal.

(c) It is a non-causal signal.

(d) It is a periodic signal.

[2015 : 2 Marks, Set-3]

21. Consider the sequence $x[n]=a^n u[n]+b^n u[n]$, where u[n] denotes the unit-step sequence and $0<|a|<|b|<1$. The region of convergence (ROC) of the z-transform of x[n] is

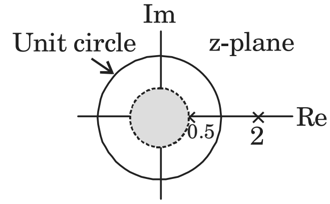
(a) $|z|>|a|$ (b) $|z|>|b|$

(c) $|z|<|a|$ (d) $|a|<|z|<|b|$

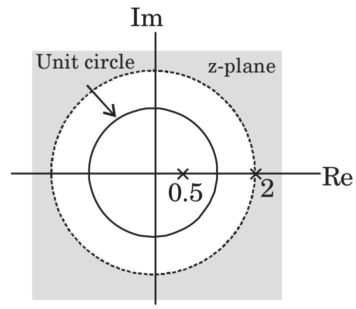
[2016 : 1 Mark, Set-1]

22. The ROC (region of convergence) of the z-transform of a discrete-time signal is represented by the shaded region in the z-plane. If the signal $x[n]=(2.0)^{|n|}, -\infty<n<+\infty$ then the ROC of its z-transform is represented by

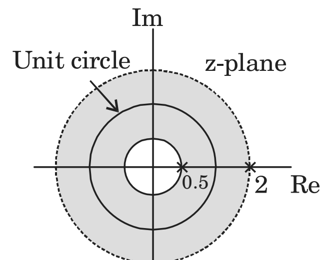
(a)



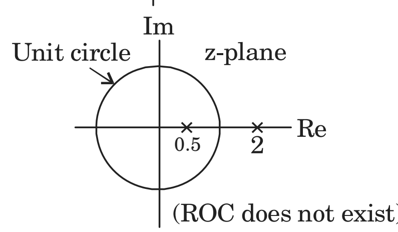
(b)



(c)



(d)



[2016 : 2 Marks, Set-3]

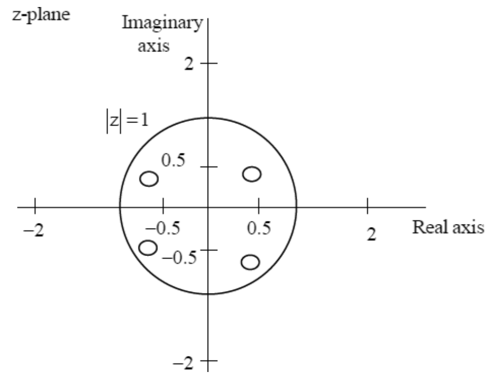
23. A discrete-time signalx[n] = $\delta[n-3]+2\delta[n-5]$ has z-transform X(z). If Y(z) = X(–z) is the z-transform of another signal y[n], then

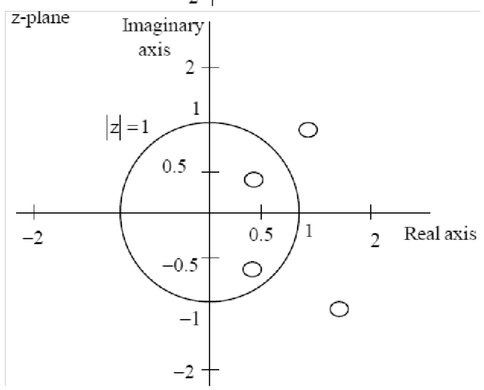
(a) y[n] = x[n] (b) y[n] = x[–n]

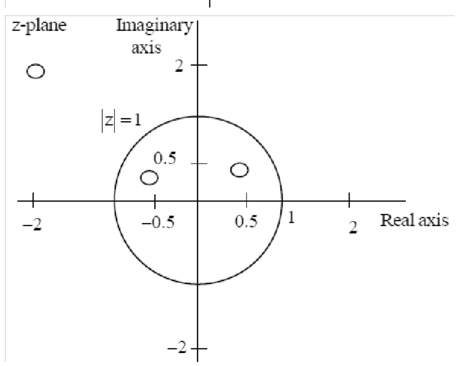
(c) y[n] = –x[n] (d) y[n] = –x[–n]

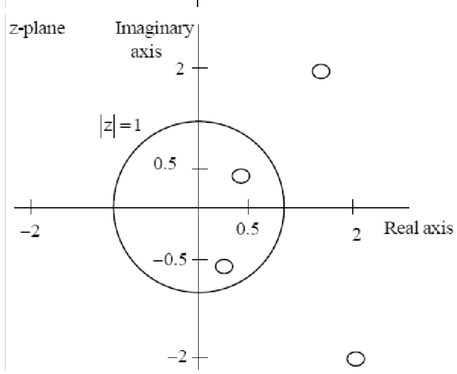
[2016 : 1 Mark, Set-3]

24. Let H(z) be the z-transform of a real-valued discrete time signal h[n]. If $P(z)=H(z)H\left(\dfrac{1}{z}\right)$ has a zero $z=\dfrac{1}{2}+\dfrac{1}{2}j$, and P(z) has a total of four zeros, which one of the following plots represents all the zeros correctly?

(a) 

(b) 

(c) 

(d) 

[2019 : 1 Mark]

25. The transfer function of a stable discrete-time LTI system is $H(z) = \dfrac{K(z - \alpha)}{(z + 0.5)}$, where K and α are real numbers. The value of α (**rounded off to one decimal place**) with $|\alpha| > 1$, for which the magnitude response of the system is constant over all frequencies, is ____.

[2020 : 2 Marks]

26. For a causal discrete-time LTI system with transfer function

$$H(z) = \frac{2z^2 + 3}{\left(z + \frac{1}{3}\right)\left(z - \frac{1}{3}\right)}$$

which of the following statements is/are true?

(a) The system is stable.

(b) The final value of the impulse response is 0.

(c) The system is a minimum phase system.

(d) The initial value of the impulse response is 2.

[2024 : 1 Mark]

INTERCONNECTION

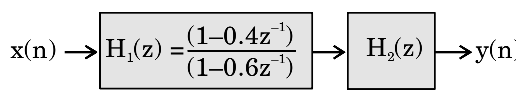
27. Two discrete time systems with impulse responses $h_1[n] = \delta[n - 1]$ and $h_2[n] = \delta[n - 2]$ are connected in cascade. The overall impulse response of the cascaded system is

(a) $\delta[n-1] + \delta[n-2]$ (b) $\delta[n-4]$

(c) $\delta[n-3]$ (d) $\delta[n-1]\,\delta[n-2]$

[2010 : 1 Mark]

28. Two systems $H_1(z)$ and $H_2(z)$ are connected in cascade as shown below. The overall output y(n) is the same as the input x(n) with a one unit delay. The transfer function of the second system $H_2(z)$ is



(a) $\dfrac{\left(1 - 0.6\,z^{-1}\right)}{z^{-1}\left(1 - 0.4\,z^{-1}\right)}$ (b) $\dfrac{z^{-1}\left(1 - 0.6\,z^{-1}\right)}{\left(1 - 0.4\,z^{-1}\right)}$

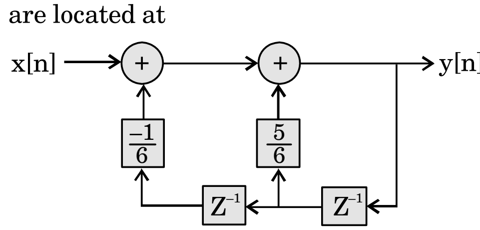
(c) $\dfrac{z^{-1}\left(1 - 0.4\,z^{-1}\right)}{\left(1 - 0.6\,z^{-1}\right)}$ (d) $\dfrac{\left(1 - 0.4\,z^{-1}\right)}{z^{-1}\left(1 - 0.6\,z^{-1}\right)}$

[2011 : 2 Marks]

29. Let $H_1(z) = (1 - pz^{-1})$, $H_2(z) = (1 - qz^{-1})^{-1}$, $H(z) - H_1(z) + rH_2(z)$. The quantities p, q, r are real numbers. Consider $p - \frac{1}{2}, q - \frac{1}{4}, |r| < 1$. If the zero of H(z) lies on the unit circle, then r = _______.

[2014 : 2 Marks, Set-3]

30. For the discrete-time system shown in the figure, the poles of the system transfer function are located at



(a) 2, 3 (b) $\frac{1}{2}, 3$

(c) $\frac{1}{2}, \frac{1}{3}$ (d) $2, \frac{1}{3}$

[2015 : 2 Mark, Set-?]

DIGITAL FILTER DESIGN

31. An FIR system is described by the system function

$$H(z) = 1 + \frac{7}{2}z^{-1} + \frac{3}{z}z^{-2}$$

The system is

(a) maximum phase. (b) minimum phase.

(c) mixed phase. (d) zero phase.

[2008: 1 Mark]

32. A system with transfer function H(z) has impulse response h(n) defined as h(2) = 1, h(3) = –1 and h(k) = 0 otherwise. Consider the following statements.

S_1 : H(z) is a low-pass filter.

S_2 : H(z) is an FIR filter.

Which of the following is correct?

(a) Only S_2 is true.

(b) Both S_1 and S_2 are false.

(c) Both S_1 and S_2 are true, and S_2 is a reason for S_1.

(d) Both S_1 and S_2 are true, but S_2 is not a reason for S_1. **[2009 : 2 Marks]**

33. For an all-pass system $H(z) = \frac{(z^{-1} - b)}{(1 - az^{-1})}$, where $|H(e^{-j\omega})| = 1$, for all ω. If Re(a) ≠ 0, lm(a) ≠ 0, then b equals

(b) a (b) a^*

(c) $\frac{1}{a^*}$ (d) $\frac{1}{a}$

[2014 : 1 Mark, Set-3]

34. Consider a four point moving average filter defined by the equation $y[n] = \sum_{i=0}^{3} \alpha_i x[n-i]$. The condition on the filter coefficients that results in a null at zero frequency is

(a) $\alpha_1 = \alpha_2 = 0; \alpha_0 = -\alpha_3$.

(b) $\alpha_1 = \alpha_2 = 1; \alpha_0 = -\alpha_3$.

(c) $\alpha_0 = \alpha_3 = 0; \alpha_1 = \alpha_2$.

(d) $\alpha_1 = \alpha_2 = 0; \alpha_0 = \alpha_3$.

[2015 : 1 Mark, Set-3]

35. A discrete-time all-pass system has two of its poles at 0.25∠0° and 2∠30°. Which one of the following statements about the system is TRUE?

(a) It has two more poles at 0.5∠30° and 4∠0°.

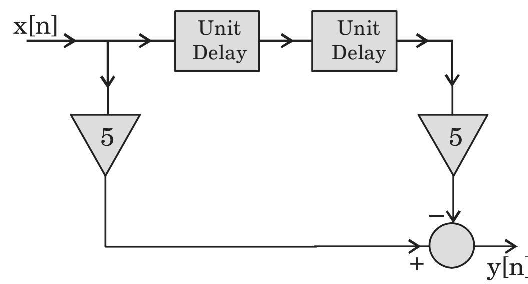
(b) It is stable only when the impulse response is two-sided.

(c) It has constant phase response over all frequencies.

(d) It has constant phase response over the entire z-plane.

[2018 : 1 Mark]

36. The direct form structure of an FIR (finite impulse response) filter is shown in the figure.



The filter can be used to approximate a

(a) low-pass filter (b) high-pass filter

(c) band-pass filter (d) band-stop filter

[2016 : 2 Marks, Set-3]

37. An LTI system with unit sample response $h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$ is a

(a) low-pass filter (b) high-pass filter

(c) band-pass filter (d) band-stop filter

[2017 : 1 Mark, Set-2]

38. It is desired to find three-tap causal filter which gives zero signal as an output to and input of the form

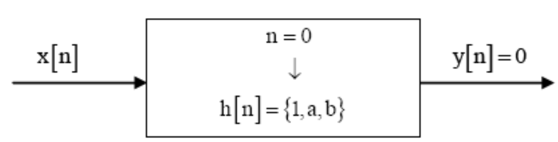
$$x[n] = c_1 \exp\left(-\frac{j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right),$$

Where c_1 and c_2 are arbitrary real numbers. The desired three-tap filter is given by

h[0] = 1, h[1] = a, h[2] = b and

h[n] = 0 for n < 0 or n > 2.

What are the values of the filter taps a and b if the output is y[n] = 0 for all n, when x[n] is as given above?



(a) a = –1, b = 1 (b) a = 0, b = 1

(c) a = 1, b = 1 (d) a = 0, b = – 1

[2017 : 2 Marks]

39. Let h[n] be length-7 discrete-time finite impulse response filter, given by

$h[0] = 4, h[1] = 3, h[2] = 2, h[3] = 1$

$h[-1] = -3, h[-2] = -2, h[-3] = -1,$

and h[n] is zero for $|n| \geq 4$. A length-3 finite impulse response [FIR] approximation g[n] of h[n] has to be obtained such that $E(h, g) = \int_{-\pi}^{\pi} \left|H\left(e^{j\omega}\right) - G\left(e^{j\omega}\right)\right|^2 d\omega$ is minimized, where $H(e^{j\omega})$ and $G(e^{j\omega})$ are the discrete-time Fourier transforms of h[n] and g[n], respectively. For the filter that minimizes E(h, g), the value of 10g[–1] + g[1], rounded off to 2 decimal places, is__________.

[2019 : 2 Marks]

ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (a)
11. (c)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (12)	**18.** (a)	**19.** (0)	**20.** (c)
21. (b)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (–2 to –2)		**26.** (a, b, d)	**27.** (c)	**28.** (b)	**29.** (–0·5)
30. (c)	**31.** (c)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (b)	**36.** (2.1)	**37.** (c)	**38.** (b)	**39.** (–27)

EXPLANATIONS

1. $$H(z) = \sum_{n=0}^{\infty} u(n) \cdot z^{-n} = \sum_{n=0}^{\infty} 1 \cdot z^{-n}$$

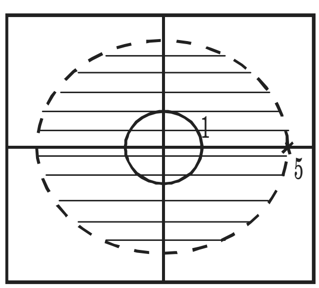
For convergence, $\left|\sum_{n=0}^{\infty} z^{-n}\right| < \infty$

or, $|z^{-1}| < 1$

or, $|z| > 1$

2. $\because \quad -a^n u[-n-1] \leftrightarrow \dfrac{z}{z-a} \qquad |z| < |a|$

$\therefore \quad -5^n u[-n-1] \leftrightarrow \dfrac{z}{z-5} \qquad |z| < |5|$



$\because$ ROC contains unit circle. Hence system is stable.

3. Output $\quad Y(z) = H(z)\, X(z)$

$= 2(z^4 + z^2 - 2z + 2 - 3z^{-4})z^{-3}$

$= 2(z + z^{-1} - 2z^{-2} + 2z^{-3} - 3z^{-7})$

Taking inverse z-transform, we have

$y(n) = 2\,[\delta(n+1) + \delta(n-1) - 2\,\delta(n-2) + 2\,\delta(n-3) - 3\delta(n-7)]$

At $n = 4$, $\quad y(4) = 0$

4. Using the following transform pair,

$$-A_k\, (d_k)^n\, u[-n-1] \overset{z}{\longleftrightarrow} \frac{A_k}{1 - d_k z^{-1}}$$

with ROC : $|z| < d_k$

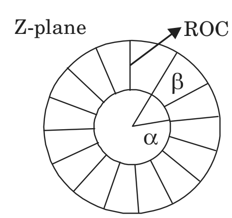
We have $\dfrac{z}{z-0.2} \overset{Z}{\longleftrightarrow} -(0.2)^n\, u[-n-1]$.

5. $$2y(z) = \alpha\, z^{-2} y(z) + \beta z^{-1} x(z) - 2x(z)$$

$$(2 - \alpha z^{-2})\, y(z) = (\beta z^{-1} - 2)\, x(z)$$

$$\frac{y(z)}{x(z)} = \frac{(\beta z^{-1} - 2)}{(2 - \alpha z^{-2})}$$

For system to be stable, the ROC should include unit circle.



$\Rightarrow \quad 2 - \alpha\, z^{-2} > 0$

$\Rightarrow \quad 2 > \alpha\, z^{-2}$

$\Rightarrow \quad z > \sqrt{\dfrac{\alpha}{2}}$

$\beta\, z^{-1} > 2$

$$\frac{\beta}{2} > z > \sqrt{\frac{\alpha}{2}}$$

Hence $|\alpha| < 2$ and $|\beta|$ for any value.

6. $x(n) = \left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{5}{6}\right)^n u[n] z^{-n} - \sum_{n=-\infty}^{\infty} \left(\frac{6}{5}\right)^n u[-n-1] \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n z^{-n} - \left(1 + \sum_{n=-\infty}^{\infty} \left(\frac{6}{5}\right)^n u(-n) z^{-n}\right)$$

$$= \sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n z^{-n+1} + \text{L} - \sum_{n=0}^{\infty} \left(\frac{6}{5}\right)^{-n} z^n$$

The first term will converge when $\left|\frac{5}{6} z^{-1}\right| < 1$

or, $|z| > \frac{5}{6}$

The second term will converge, when, $\left|\frac{6}{5} z^{-1}\right| > 1$

or, $|z| < \frac{6}{5}$

Thus, the region of convergence is

$$\frac{5}{6} < |z| < \frac{6}{5}$$

7. For **right sided** exponential sequence,

$$x_1[n] = a^n u[n]$$

$$X_1[z] = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

Convergence requires that $\sum_{n=-0}^{\infty} |az^{-1}|^n < \infty$

This is possible only or, if $|az^{-1}| < 1$

or, $|z| > |a|$

For **left-sided** exponential sequence

$$x_2[n] = b^{-n} u[-n-1]$$

$$X_2[z] = \sum_{n=-\infty}^{\infty} -b^n u[-n-1] z^{-n} = 1 - \sum_{n=0}^{\infty} (b^{-1}z)^n$$

$X_2(n)$ converges only if $|b^{-1}z| < 1$ or, $|z| < |b|$

Hence, $x_1[n] + x_2[n]$ have ROC given by

$$\frac{1}{3} < |z| < \frac{2}{3}, a = \frac{1}{3}, b = \frac{2}{3}$$

For, $x_1[n] - x_2[n]$;

$$X_3[z] = X_1[z] - X_2[z]$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n - 1 + \sum_{n=0}^{\infty} (b^{-1}z)^n$$

$X_2[z]$ will be converge for $|az^{-1}| < 1$ and $|b^{-1}z| < 1$

or, $|z| < |b|$ and $|z| > |a|$

ROC remains same $\frac{1}{3} < |z| < \frac{2}{3}$

8. For left- handed signal,

$$X[z] = -\sum_{n=-\infty}^{\infty} a^n u(-n-1) z^{-n} = -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$= -\sum_{n=1}^{\infty} (a^{-1}z)^n = 1 - \sum_{n=0}^{\infty} (a^{-1}z)^n$$

as $\sum_{n=0}^{\infty} (a^{-1}z)^n = 1 + a^{-1}z + (a^{-1}z)^2 + \ldots = \frac{1}{1 - a^{-1}z}$

then, $X[z] = 1 - \frac{1}{1 - a^{-1}z} = \frac{1}{1 - az^{-1}}$

and ROC is given by $|z| < |a|$

Here $a = 2$,

then, ROC: $|z| < 2$ includes unit circle

Hence, $x[n] = -0.5[z]^n u[-n-1]$

and $x[0] = 0$

9. $x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$

Let, $x_1(n) = \left(\frac{1}{3}\right)^n u[n]$

$$x_1[z] = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n)_z^{-n}$$

$$= -\sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n = \frac{1}{1 - \frac{1}{3} z^{-1}}$$

$X_1[z]$ will converge where $\left|\frac{1}{3} z^{-1}\right| < 1$ or, $|z| > \frac{1}{3}$

and, $x_2[n] = -\left(\frac{1}{2}\right)^n u(-n-1)$

$$X_2[z] = -\sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(-n-1)_z^{-n} - \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= -\sum_{n=1}^{\infty} \left(\frac{1}{2} z^{-1}\right)^{-n} = -\sum_{n=1}^{\infty} (2z)^n$$

$$= 1 - \sum_{n=0}^{\infty} (2z)^n$$

For convergence of $X_2(z)$, $|2z| < 1$ or $|z| < \frac{1}{2}$

Hence, ROC for $x(n)$ should be in the range of $\frac{1}{3} < |z| < \frac{1}{2}$

10. From, z-transform property of

$$z^{-m} \leftrightarrow \delta[n-m]$$

We has $X(z) \leftrightarrow 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$

11. A discrete-time LT1 system is B1B0 (Bounded input bounded output) stable if and only if its impulse response $h[n]$ is absolutely summable, that is

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Now, $\quad H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$

let z $= e^{i\Omega}$ so that

$|z| = |e^{-j\Omega}| = 1$ then

$$|H(e^{i\Omega})| = \frac{N_0}{2} = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

It can be inferred that if the system is stable, then $H(z)$ converges for $z = e^{i\Omega}$. hence, for a stable discrete-time LT1 system, the ROC of $H(z)$ must contain the unit circle $|z| = 1$.

Further, for the system having impulse response

$$h[n] = \alpha^n u[n],\ H(z) = \frac{z}{z-\alpha},\quad |z| > |\alpha|$$

For $|z| > |\alpha|$, ROC of $H(z)$ includes $z = \infty$, therefore $h(n)$ is a causal sequence and thus the system is causal.

Now, Given, $\quad H(z) = \dfrac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

$$= \frac{1}{1 - \frac{1}{4}Z^{-1}} + \frac{1}{1 - \frac{1}{2}Z^{-1}}$$

For, S1 : ROC, $|z| > \frac{1}{2}$, $H(z)$ contain unit circle and includes $z = \infty$, the system is stable and causal.

For, S2 : ROC $|z| < \frac{1}{4}$, $H(z)$ does not contain unit circle and excludes $z = \infty$, the system is neither stable nor causal.

For, S3 : ROC $\frac{1}{4} < |z| < \frac{1}{2}$, $H(z)$ does not contain unit circle and excludes $z = \infty$, the system is neither stable nor causal.

12. $\quad x[n] = (1/3)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$

For $(1/3)^{|n|}$, $\quad$ ROC is $\frac{1}{3} < |z| < 3$

For $(1/2)^{|n|} u[n]$, $\quad$ ROC is $|z| > \frac{1}{2}$

Thus common ROC is $\frac{1}{2} < |z| < 3$

13. (c)

14. $\quad x(n) = \underbrace{\left[-\frac{1}{9}\right]4(x)}_{\text{Right side Signal}} - \underbrace{\left(\frac{1}{3}\right)^n 4(-n-1)}_{\text{Left side signal}}$

ROC is $|z| > \frac{1}{9}$

ROC is $|z| < \frac{1}{3}$

So ROC is $\frac{1}{3} > |z| > \frac{1}{9}$

15. Given $\quad x[n] = x[-n]$

$\Rightarrow \quad x(z) = x(z^{-1})$

[Time reversal property in z – transform]

$\Rightarrow$ if one zero is $0.5 + j0.25$

then other zero will be $\dfrac{1}{0.5 + j0.25}$

16. Given system equation as

$$y[n] = \alpha y[n-1] + \beta x[n]$$

$$\Rightarrow \quad \frac{y(z)}{x(z)} = \frac{\beta}{1 - \alpha z^{-1}}$$

$$\Rightarrow \quad H(z) = \frac{\beta}{1 - \alpha z^{-1}}$$

$h[n] = \beta(\alpha)^h u[n]$ [causal system]

Also given that $\sum_{h=0}^{\infty} h[n] = 2$

$$\beta\left[\frac{1}{1-\alpha}\right] = 2$$

$$1 - \alpha = \frac{\beta}{2}$$

$$\alpha = 1 - \frac{\beta}{2}$$

17. $X(z) = \dfrac{1}{(1-2z^{-1})^2} = \dfrac{1}{(1-2z^{-1})}\dfrac{1}{(1-2z^{-1})}$

$$x[n] = 2^n u[n] * 2^n u[n]$$

$$x[n] = \sum_{k=0}^{n} 2^k . 2^{(n-k)}$$

$$\Rightarrow \quad x[2] = \sum_{k=0}^{2} 2^k . 2^{(2-k)} \quad 2^0.2^2 + 2^1.2^1 + 2^2.2^0$$

$$= 2^0.2^2 + 2^1.2^1 + 2^2.2^0$$

$$= 4 + 4 + 4 = 12$$

18. (a)

19. $\quad y[n] = \sum_{m=0}^{n} x[m]$

According to accumulation property of z-transform,

$$Y[z] = \frac{X(z)}{(1 - z^{-1})}$$

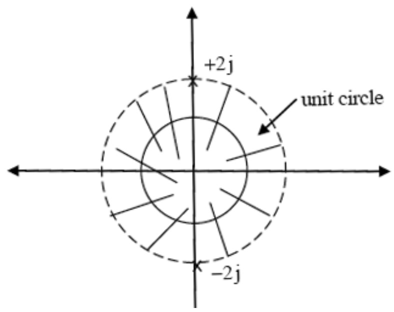
$$\Rightarrow \quad \frac{2}{z(z-1)} = \frac{zX(z)}{(z-1)}$$

$\therefore \quad X[z] = \dfrac{2z^{-2}}{(z-1)} = \dfrac{2z^{-3}}{(1-z^{-1})}$

$\therefore \quad x[n] = 2u[n-3]$

thus $\quad x[2] = 0$

20. Since $x[n]$ in absolutely summable thus its ROC must include unit circle.



Thus ROC must be inside the circling radius 2.

$x[n]$ must be a non-causal signal.

21. Given sequence

$$x(n) = (a)^n x(n) + (b)^n x(n),$$

Also given $\quad 0 < |a| < |b| < 1$

$\therefore$ The region of convergence (ROC)

$$= (|z| > |a|) \cap (|z| > |b|)$$

$$= |z| > |b|$$

22. $x(y) = (2)^n u(n) + \left(\dfrac{1}{2}\right)^n u(-n-1)$

ROC = $(|z| > 2) \cap (|z| > ½) = \phi$

So, the ROC of z-tansform is null.

23. Here $\quad (a)^n x(n) \leftrightarrow X(z/a)$

$$a = -1$$

$$(-1)^n x(n) \leftrightarrow X(-z)$$

but $\quad x(n) = \delta[n-3] + 2\delta[n-5]$

$y(n) = (-1)^n x(n)$

$= (-1)^n [\delta(n-3) + 2\delta[n-5]$

$\therefore \quad y(n) = -\delta(n-3) - 2\delta(n-5)$

$= -x(n)$

Hence, the value of signal $y(n)$ is $-x(n)$.

24. It is given that H(z) is z-transform of a real-valued signal h(n).

$P(z) = H(z)H\left(\dfrac{1}{z}\right)$ and P(z) has 4 zeros $\Rightarrow$ P(z) are,

sum of zeros of H(z) and zeros of $H\left(\dfrac{1}{z}\right)$.

If $z_1 = \dfrac{1}{2} + j\dfrac{1}{2}$ is one zero then there must be a zero at $z_1^* = \dfrac{1}{2} - j\dfrac{1}{2}$

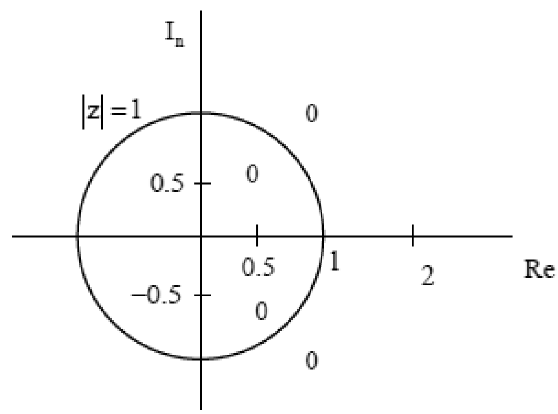
Let z_1, z_1^* represent zeros of H(z) then the zeros of $H\left(\dfrac{1}{z}\right)$ will be $\dfrac{1}{z_1}$ and $\left(\dfrac{1}{z_1}\right)^*$

$$\frac{1}{z_1} = \frac{1}{\frac{1}{2} + j\frac{1}{2}} = 1 - i$$

$$\left(\frac{1}{z_1}\right)^* = 1 + j$$

Hence zeros of P(z) are $\left(\dfrac{1}{2} \pm j\dfrac{1}{2}\right)$ and $(1 \pm j)$ or

$[0.707\ \underline{|\pm 45°}\,]$ and $[\sqrt{2}\ \underline{|\pm 45°}\,]$



Even option D looks like similar but in option B, the zeros that are outside the unit circle have real part 2, but we need 1.

25. As given that T/f of LTI system is:

$$H(z) = \frac{K(z-\alpha)}{z+0.5}, |\alpha| > 1$$

$$|H(e^{j\omega})| = 1\ \forall\ \omega$$

For an all pass filter poles & zeros are reciprocal to each other.

To get constant magnitude for all frequency pole located at z = – 0.5

$\therefore \quad$ zero is located at $= \dfrac{1}{\text{Pole}} = \dfrac{1}{-0.5} = -2$

$\therefore \quad$ zero $= \alpha = -2$

So, $H(z) = \dfrac{k(z+2)}{z+0.5}$

26. As given that ;

$$H(z) = \frac{2z^2+3}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)}$$

So, the system is non-minimum phase system.

Since zero are lying outside the unit circle.

Initial value :

$$h(0) = \lim_{z\to\infty} \frac{\left(2z^2+3\right)}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)}$$

$$= \lim_{z\to\infty} \frac{z^2\left[2+\frac{3}{z^2}\right]}{z^2\left[1+\frac{1}{3z}\right]\left[1-\frac{1}{3z}\right]}$$

$$= \lim_{z\to\infty} \frac{\left[2+\frac{3}{z^2}\right]}{\left[1+\frac{1}{3z}\right]\left[1-\frac{1}{3z}\right]}$$

$$= \frac{2}{1\times 1} = 2$$

Final value :

$$h(\infty) = \lim_{z\to 1}\left(1-z^{-1}\right)\cdot H(z)$$

$$h(\infty) = \lim_{z\to 1}\left(1-z^{-1}\right)\frac{2z^2+3}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)} = 0$$

Now, from H(z) given system :

So, $\quad z+\frac{1}{3}=0,\ z-\frac{1}{3}=0$

$$z = -\frac{1}{3},\ z=\frac{1}{3}$$

$$|z| = \frac{1}{3} = 0.33$$

Hence, the system is stable.

If, |pole| < 1 then pole are lying inside the unit circle.

27.

$$h_1[n] = \delta(n-1) \qquad h_1(z) = z^{-1}$$

$$h_2(n) = \delta(n-2) \qquad h_2(z) = z^{-2}$$

Hence in cascade, overall z-transform of impulse response,

$$H(z) = h_1(z)\cdot h_2(z)$$

$$= z^{-1}\cdot z^{-2} = z^{-3}$$

$$\therefore \quad h(n) = \delta(n-3)$$

28. Overall transfer function $= z^{-1}$

(since unit delay T.F $= z^{-1}$)

$$H_1(z)\,H_2(z) = z^{-1}$$

$$H_2(z) = \frac{z^{-1}}{H_1(z)} = z^{-1}\frac{(1-0.6z^{-1})}{(1-0.4z^{-1})}$$

29.

$$H_1(z) = (1-Pz^{-1})^{-1}$$

$$H_2(z) = (1-qz^{-1})^{-1}$$

$$H(z) = \frac{1}{1-Pz^{-1}} + r\frac{1}{(1-qz^{-1})}$$

$$= \frac{1-qz^{-1}+r(1-Pz^{-1})}{(1-Pz^{-1})(1-Pz^{-1})}$$

$$= \frac{(1+r)-(q+rp)z^{-1}}{(1-Pz^{-1})(1-Pz^{-1})}$$

zero of $\quad H(z) = \dfrac{q+rp}{1+r}$

Since zero is existing on unit circle

$$\Rightarrow \quad \frac{q+rp}{1+r} = 1$$

$$\text{or} \quad \frac{q+rp}{1+r} = -1$$

$$\frac{-\frac{1}{4}+\frac{r}{2}}{1+r} = 1$$

$$\text{or} \quad \frac{-\frac{1}{4}+\frac{r}{2}}{1+r} = -1$$

$$-\frac{1}{4}+\frac{r}{2} = 1+r$$

$$\text{or} \quad -\frac{1}{4}+\frac{r}{2} = -1-r$$

$$\Rightarrow \quad r = -\frac{5}{2}$$

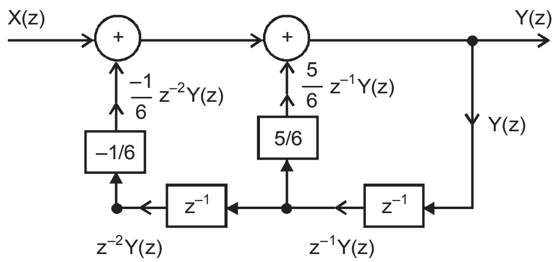
$$\Rightarrow \quad \frac{r}{2} = -\frac{5}{4}$$

$$\text{or} \quad \frac{3}{4} = \frac{-3r}{2}$$

$$r = -\frac{1}{2}$$

$$\Rightarrow \quad r = -0.5$$

$$r = -\frac{5}{2} \text{ is not possible}$$

30. 

$$X(z) - \frac{1}{6}z^{-2}Y(z) + \frac{5}{6}z^{-1}Y(z) = Y(z)$$

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = X(z)$$

$$Y(z)\left\{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right\} = X(z)$$

T.F. of the system, $H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right)}$

$$H(z) = \frac{z^2}{\left(z^2 - \frac{5}{6}z + \frac{1}{6}\right)}$$

$$H(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$

Pole location :

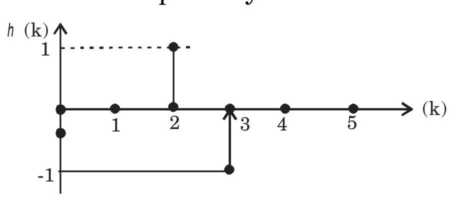
$z - \frac{1}{2} = 0 \quad \Rightarrow z = \frac{1}{2}$

$z - \frac{1}{3} = 0 \quad \Rightarrow z = \frac{1}{3}$

31. Minimum phase system has all zeros inside unit circle maximum phase system has all zeros outside unit circle mixed phase system has some zero outside unit circle and some zeros inside unit circle.

for $\qquad H(s) = 1 + \frac{7}{2}z^{-1} + \frac{3}{2}z^{-2}$

One zero is inside and one zero outside unit circle hence mixed phase system

32. 

$\therefore \quad H(e^{j\omega}) = e^{-j2\omega} - e^{-j3\omega}$

So, it is FIR high pass filter.

33. For an all pass system,

$$\text{pole} = \frac{1}{\text{zero}^*}$$

or $\qquad \text{zero} = \frac{1}{\text{pole}^*}$

$$\text{pole} = a$$

$$\text{zero} = \frac{1}{b}$$

$\Rightarrow \qquad \frac{1}{b} = \frac{1}{a^*}$

or $\qquad b = a^*$

34. Given $y[n] = \sum_{i=0}^{2} \alpha_i x(n-i)$

$\Rightarrow \; y[n] = \alpha_0 x[n] + \alpha_1 x[n-1] + \alpha_2 x[n-2] + \alpha_3 x[n-3]$

Getting a null at zero frequency implies that given filter can be high pass filter but it cannot be low pass filter.

High pass filter is possible if we have negative coefficients.

Let say, $\qquad \alpha_1 = \alpha_2 = 0,\ \alpha_0 = -\alpha_3$

$\Rightarrow \qquad y[n] = -\alpha_3 x[n] + a_3 x[n-3]$

$$H(z) = -\alpha_3[1 - z^{-3}]$$

$\Rightarrow \qquad H(e^{j\Omega}) = -\alpha_3[1 - e^{-j3\Omega}]$

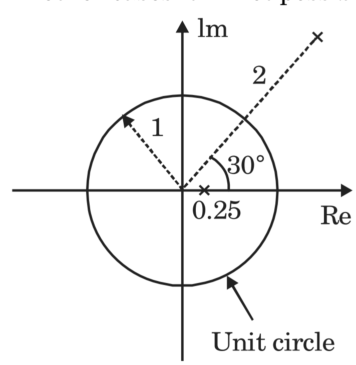
$$= -\alpha_3 e^{\frac{-j3\Omega}{2}}\left[\frac{e^{j\frac{3\Omega}{2}} - e^{-j\frac{3\Omega}{2}}}{2j}\right] \times 2j$$

$$= -\alpha_3 2j\sin\left(\frac{3\Omega}{2}\right).e^{-j\frac{3\Omega}{2}}$$

$$= -\alpha_3 2.\sin\frac{3\Omega}{2}.e^{-j\frac{3\Omega}{2}}.e^{j\frac{\pi}{2}}$$

$\Rightarrow \qquad H(e^{j\Omega})|_{\Omega=0} = 0$

In other cases it in not possible.

35. 

The ROC should encircle unit circle to make the system stable. From the given pole pattern it is clear that, to make the system stable, the ROC should be two-sided. Thus impulse response for the system should be two-sided.

36. Given : $h[n]=\frac{1}{3}\delta[n]+\frac{1}{3}\delta[n-1]+\frac{1}{3}.\delta[n-2].$

$$H\left(e^{j\omega}\right)=\frac{1}{3}f^{j\omega}[1+2\cos\omega]$$

$$H\left(e^{j\omega}\right)=0\Rightarrow 1+2\cos\omega_o=0$$

$$\Rightarrow \cos\omega_o=\frac{-1}{2};\omega_o=\frac{2\pi}{3}=2.10\text{rad}.$$

37. Given : h [n] = 5 [n] – 7δ [n – 1] + 7δ [n – 3] – 5δ [n – 4].

$H(e^{j\omega}) = 5 - 7 \,.\, e^{-j\omega} + 7e^{-3j\omega} - 5e^{-4j\omega}$

Now, for $\omega = 0$, $H(e^{j0}) = 5 - 7 \times 1 + 7 \times 1 - 5 \times 1 = 0$.

For $\omega = \pi$, $H(ej\pi) = 5 - 7(-1) + 7(-1) + 5(-1) = 5 + 7 - 7 - 5 = 0$.

System is attenuating low and high frequencies whereas passing the mid frequencies. So, it is a bandpass filter.

38. It is given that

h(n) = [1, a, b]

and $x(n) = C_1e^{-j\frac{\pi}{2}n}+C_2e^{j\frac{\pi}{2}n}$

$\Rightarrow y(n)=0$

Now If $h(n)=[1,a,b]$

$\therefore\quad H\left(e_{j\omega}\right)=1+ae^{-j\omega}+be^{-j2\omega}$

$\rightarrow$ When $x(n)=C_1e^{-j\frac{\pi}{2}n}+C_2e^{j\frac{\pi}{2}n}$ then expression of y(n)

Since the input x(n) contain 2 frequencies $\pm\frac{\pi}{2}$,

Let evaluate $\left|H\left(e^{j\omega}\right)\right|$ at f nil 2 frequency

$$H\left(e^{j-\pi/2}\right)=1+ae^{-j\left(\frac{-\pi}{2}\right)}+be^{-j2\left(\frac{-\pi}{2}\right)}$$

$$=1+ae^{+j\pi/2}+be^{j\pi}$$

$$= 1 + [a(j)] + [b(-1)]$$

$$= (1 - b) + j(a)$$

$\therefore\qquad H(e^{j\pi/2}) = (1 - b) - ja$

$$\left|H\left(e^{j\pi/2}\right)\right|=\left|H\left(e^{-j\frac{\pi}{2}}\right)\right|=\sqrt{(1-b)^2+a^2}$$

So the expression of y(n) is

$$y(n)=\left[(1-b)^2+a^2\right]^{1/2}C_1e^{-j\left(\frac{1}{2}n+\phi_1\right)}$$

$$+\left[(1-b)^2+a^2\right]^{1/2}C_2e^{j\left(\frac{\pi}{2}n+\phi_2\right)}$$

for y(n) = 0

$$\Rightarrow\ k=\sqrt{(1-b)^2+a^2}=0$$

Now from option

from option (a) a = –1, b = 1, then

$k=\sqrt{0^2+1^2}=1$(not correct)

from option (b) a = 0, b = 1 then

$k=\sqrt{0^2+0^2}=0$(correct)

from option (c) a = 1, b = 1, then

$k=\sqrt{0^2+1^2}=1$(not correct)

from option (d) a = 0, b = 1, then

$k=\sqrt{2^2+0^2}=2$(not correct)

39. It is given that

h(n) = [–1, –2, –3, 4, 3, 2, 1] ...(i)

$g(n) = [a,\ \underset{\uparrow}{b},\ c]$ ...(ii)

It is mentioned that

$E(h,g)]\int_{-x}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2d\omega$, is minimised,

If h(n) and g(n) represent IDTFT of $H(e^{j\omega})$, $G(e^{j\omega})$

then E(h, g) $=2\pi\sum\left|h(n)-g(n)\right|^2$ (By Parseval theorem)

$h(n) \leftrightarrow H(e^{j\omega})$

$g(n) \leftrightarrow G(e^{j\omega})$

Now, $h(n) - g(n) \leftrightarrow H(e^{j\omega}) - G(e^{j\omega})$

Energy of

$$\left[h(n)-g(n)\right]=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2d\omega$$

$\Rightarrow 2\pi$ energy of

$$\left[h(n)-g(n)\right]=\int_{-\pi}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2d\omega$$

$$\Rightarrow E(h,g)=\int_{-x}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2d\omega$$

$$=2\pi\sum_{n=\infty}^{\infty}\left|h(n)-g(n)\right|^2$$

We want to minimize E (h, g)

Using equation (i) and equation (ii) we get

h(n) – g(n) = [–1, –2, –3 – a, 4 – b, 3 – c, 2, 1]

$\therefore \quad E(h,g) = 2\pi \sum |h(n) - g(n)|^2 = 2\pi \sum [h(n) - g(n)]^2$

$\therefore \quad E(h, g) = 2\pi[(-1)^2 + (-2)^2 + (-3 - a)^2 + (4 - b)^2 + (3 - c)^2 + 2^2 + 1^2]$

$= 2\pi[10 + (-3 - a)^2 + (4 - b)^2 + (3 - 1)^2]$

To minimize the value of E(h,g)

Equate, $-3 - a = 0 \Rightarrow a = -3$

and $4 - b = 0 \Rightarrow b = 4$

and $3 - c = 0 \Rightarrow c = 3$

$\rightarrow g(n) = [a, b, c] = [-3, 4, 3]$

So $10g(-1) + g(1) = 10a + c = (10(-3) + 3) = -30 + 3 = -27$

7
CHAPTER

DTFT, DFT & FFT

FOURIER TRANSFORM OF DISCRETE-TIME SIGNAL

1. A Fourier transform pair is given by

$$\left(\frac{2}{3}\right)^n u(n+3) \xleftrightarrow{\text{F.T.}} \frac{Ae^{+j6\pi t}}{1-\left(\frac{2}{3}\right)e^{-j2\pi f}},$$

when u[n] denotes the unit step sequence, the values of A is _______.

[2004 : 1 Mark]

2. Let $x(n)=\left(\frac{1}{2}\right)^n u(n)$, $y(n) = x^2(n)$ and $y(e^{j\omega})$ be the Fourier transform of y(n). Then $Y(e^{j0})$ is

(a) $\frac{1}{4}$ (b) 2

(c) 4 (d) $\frac{4}{3}$

[2005 : 1 Mark]

3. The Fourier transform of y(2n) will be

(a) $e^{-j2\omega}$ | $\cos 4\omega + 2\cos 2\omega + 2]$

(b) $[\cos 2\omega + 2\cos\omega + 2]$

(c) $e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$

(d) $e^{\frac{-j\omega}{2}}[\cos 2\omega + 2\cos\omega + 2]$

[2005 : 2 Marks]

4. A signal $x(n) = \sin(\omega_0 n + f)$ is the input to a linear time-invariant system having a frequency response $H(e^{j\omega})$. If ihe output of the system $Ax(n - n_0)$, then the most general form of $\angle H(e^{j\omega})$ will be

(a) $-n_0\omega_0 + \beta$ for any arbitrary real β

(b) $-n_0\omega_0 + 2\pi k$ for any arbitrary integer k.

(c) $n_0\omega_0 + 2\pi k$ for any arbitrary integer k.

(d) $-n_0\omega_0\phi$ **[2005 : 2 Marks]**

5. Consider the signal

$x[n] = 6\delta[n + 2] + 3\delta[n + 1] + 8\delta[n] + 7\delta[n - 1] + 4\delta[n - 2]$. If $X(e^{j\omega})$ is the discrete-time Fourier transform of x[n],

then $\frac{1}{\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\sin^2(2\omega)d\omega$ is equal to ______

[2016 : 2 Marks, Set-1]

6. Let h[n] be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$$h[0]=\frac{1}{3};\ h[1]=\frac{1}{3};\ h[2]=\frac{1}{3}$$

and h[n] = 0 for n < 0 and n > 2.

Let $H(\omega)$ be the discrete-time Fourier transform (DTFT) of h[n], where to is the normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega_0 < \pi$, the value of ω_0 (in radians) is equal to _______.

[2017 : 2 Marks, Set-1]

7. Let $X[k] = k + 1$, $0 \le k \le 7$ be 8-point DFT of a sequence x[n], where $X[k]=\sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$. The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$ is _______.

[2018 : 2 Marks]

8. The output y[n] of a discrete-time system for an input x[n] is

$$y[n] = \max_{-\infty\le k\le n} |x[k]|$$

The unit impulse response of the system is

(a) 0 for all n

(b) 1 for all n

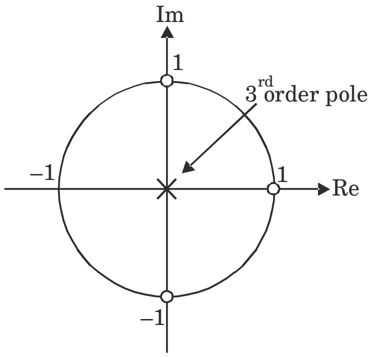
(c) unit step signal u[n]

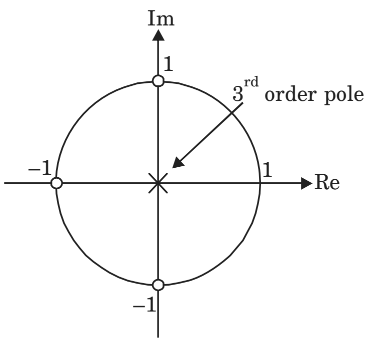
(d) unit impulse signal $\delta[n]$

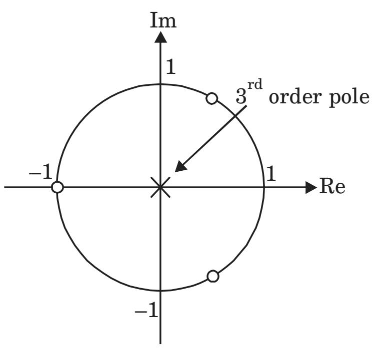
[2020 : 1 Mark]

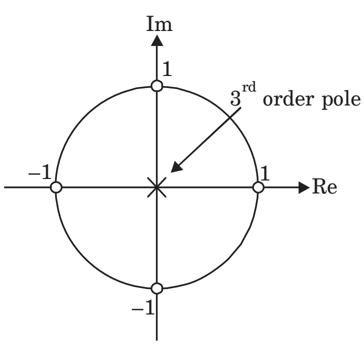
9. Which one of the following pole-zero plots corresponds to the transfer function of an LTI system characterized by the input-output difference equation given below?

$$y[n] = \sum_{k=0}^{3} (-1)^k x[n-k]$$

(a) 

(b) 

(c) 

(d) 

[2020 : 1 Mark]

10. For a unit step input $u[n]$, a discrete-time LTI system produces an output signal $(2\delta[n+1]+\delta[n]+\delta[n-1])$. Let $y[n]$ be the output of the system for an input $\left(\left(\frac{1}{2}\right)^n u(n)\right)$. The value of $y[0]$ is _________

[2021 : 2 Marks]

DISCRETE-TIME FOURIER TRANSFORM

11. A 5-point sequence x[n] is given as

$x[-3] = 1, x[-2] = 1, x[-1] = 0, x[0] = 5, x[1] = 1.$ Let $X(e^{j\omega})$ denote the discrete-time Fourier transform of x[n]. The value of $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$

(a) 5 (b) 10π

(c) 16π (d) $5 + j10\pi$

[2007 : 2 Marks]

12. {a(n)} is a real-valued periodic sequence with a period N. x(n) and X(k) form N-point Discrete Fourier Transform (DFT) pairs. The DFT Y(k) of the sequence

$$y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)x(n+r) \text{ is}$$

(a) $|X(k)|^2$

(b) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X'(k+r)$

(c) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X(k+r)$

(d) 0 **[2008 : 2 Marks]**

13. The 4-point discrete Fourier Transform (DFT) of a discrete time sequence {1, 0, 2, 3} is

(a) {0, –2 + 2j, 2, –2 – 2j}.

(b) {2, 2 + 2j, 6, 2 – 2j}.

(c) {6, 1 – 3j, 2, 1 + 3j}.

(d) {6, –1 + 3j, 0, –1 – 3j}.

[2009 : 2 Marks]

14. The first five points of the 8-point DFT of a real valued sequence are 5, $1-j3$, 0, $3-j4$ and $3 + j4$. The last two points of the DFT are respectively

(a) 0, $1-j3$ (b) 0, $1+j3$

(c) $1+j3$, 5 (d) $1-j3$, 5

[2011 : 2 Marks]

15. The DFT of a vector [a b c d] is the vector [α β γ δ]. Consider the product

$$[p\ q\ r\ s] = [a\ b\ c\ d]\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of the vector [p q r s] is a scaled version of

(a) $[\alpha^2\ \beta^2\ \gamma^2\ \delta^2]$

(b) $[\sqrt{\alpha}\ \ \sqrt{\beta}\ \ \sqrt{\gamma}\ \ \sqrt{\delta}]$

(c) $[\alpha+\beta\ \ \ \beta+\delta\ \ \ \delta+\gamma\ \ \ \gamma+\alpha]$

(d) $[\alpha\ \ \beta\ \ \gamma\ \ \delta]$

[2013 : 2 Marks]

16. The N-point DFTof a sequence x[n], $0 \le n \le N-1$ is given by

$$X[K] = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nK}, 0 \le K \le N-1$$

Denote this relation as X = DFT(x). For N = 4, which one of the following sequences satisfies DFT (DFT (x)) = x.

(a) x = [1 2 3 r] (b) x = [1 2 3 2]

(c) x = [1 3 2 2] (d) x = [1 2 2 3]

[2014 : 2 Marks, Set-4]

17. Two sequences [a, b, c] and [A, B, C] are related as,

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ where, } W_3 = e^{i\frac{2\pi}{3}}.$$

If another sequence [p, q, r] is derived as,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{1} & W_3^{2} \\ 1 & W_3^{2} & W_3^{4} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^{2} & 0 \\ 0 & 0 & W_3^{4} \end{bmatrix}\begin{bmatrix} A/3 \\ B/3 \\ C/3 \end{bmatrix}$$

then the relationship between the sequences [p, q, r] and [a, b, c] is

(a) [p, q, r] = [b, a, c]

(b) [p, q, r] = [b, c, a]

(c) [p, q, r] = [c, a, b]

(d) [p, q, r] = [c, b, a]

[2015 : 2 Marks, Set-1]

18. Consider two real sequences with time-origin marked by the bold value,

$x_1[n] = \{1, 2, 3, 0\}$, $x_2[n] = \{1, 3, 2, 1\}$.

Let $X_1(k)$ and $X_2(k)$ be 4-point DFTs of $x_1[n]$ and $x_2[n]$, respectively.

Another sequence $x_3[n]$ is derived by taking 4-point inverse DFT of $X_3(k) = X_1(k)X_2(k)$. The value of $x_3[2]$ is _______.

[2015 : 2 Marks, Set-2]

19. The Discrete Fourier Transform (DFT) of the 4-point sequence

x[n] = {x[0], x[1], x[2], x[3]} = {3, 2, 3, 4) is

X[k] = {X[0], X[1], X[2], X[3]} = {12, 2j, 0, –2j}.

If $X_1[k]$ is the DFT of the 12-point sequence

$x_1[n]$ = {3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0), the value

of $\left|\frac{X_1(8)}{X_1[11]}\right|$ is _______.

[2016 : 2 Marks, Set-2]

20. A finite duration discrete time signals x[n] is obtained by sampling the continuous-time signal $x(t) = \cos(200\pi t)$ of sampling instants $t = \frac{n}{400}$, n = 0, 1, 2, ... 7. The 8-point discrete Fourier transform (DFT) of x[n] is defined as

$$X[k] = \sum_{n=0}^{7} x[n]e^{-j\frac{\pi kn}{4}}, k = 0, 1, ..., 7$$

Which one of the following statements is TRUE?

(a) All X[k] are non-zero

(b) Only X[4] is non-zero

(c) Only X[2] and X[6] are non-zero

(d) Only X[3] and X[5] are non-zero

[2020 : 2 Marks]

21. Consider two 16-point sequences $x[n]$ and $h[n]$. Let the linear convolution of $x[n]$ and $h[n]$ be denoted by $y[n]$, while $z[n]$ denotes the 16-point inverse discrete Fourier transform (IDFT) of the product of the 16-point DFTs of $x[n]$ and $h[n]$. The value(s) of k for which $z[k] = y[k]$ is/are

(a) k = 0, 1, 2, ..., 15

(b) $k = 0$

(c) $k = 15$

(d) $k = 0$ and $k = 15$

[2021 : 1 Mark]

22. Consider the signals $x[n] = 2^{n-1} u[-n+2]$ and $y[n] = 2^{-n+2} u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$$

(*rounded off to one decimal place*) is ________

[2021 : 2 Marks]

23. For a vector $\bar{x} = [x[0], x[1], \dots, x[7]]$, the 8-point discrete Fourier transform (DFT) is denoted by $\bar{X} = DFT(\bar{x}) = [x[0], x[1], \dots, x[7]]$, where

$$X(k) = \sum_{n=0}^{7} x[n]\exp\left(-j\frac{2\pi}{8}nk\right).$$

Here, $j = \sqrt{-1}$. If $\bar{x} = [1,0,0,0,2,0,0,0]$ and $\bar{y} = DFT(DFT(\bar{x}))$, then the value of y[0] is ________ (rounded off to one decimal place).

[2022 : 2 Marks]

24. The relationship between any N-length sequence x[n] and its corresponding N-point discrete Fourier transform X[k] is defined as

X[k] = F{x[n]}.

Another sequence y[n] is formed as below

y[n] = F{F{F{F{x[n]}}}}.

For the sequence x[n] = {1, 2, 1, 3}, the value of Y[0] is _______. **[2024 : 2 Marks]**

FAST FOURIER TRANSFORM

25. For an N-point FFT algorithm with $N = 2^m$, which one of the following statements is TRUE?

(a) It is not possible to construct a signal flow graph with both input and output in normal order

(b) The number of butterflies in the mn state is $\frac{N}{m}$

(c) In-place computation requires storage of only 2N node data

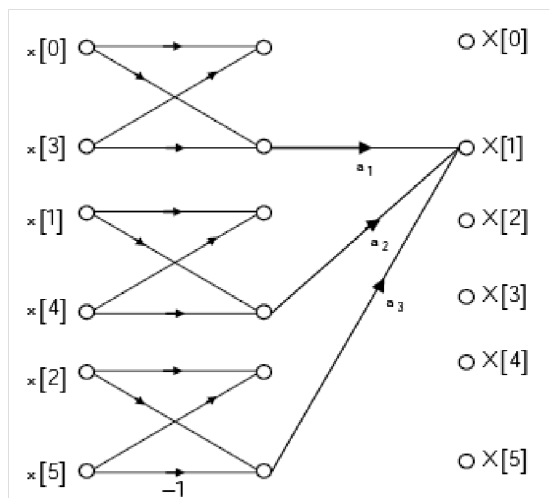
(d) Computation of a butterfly requires only one complex multiplication

[2010 : 1 Mark]

26. A continuous-time speech signal $x_a(t)$ is sampled at a rate of 8 kHz and the samples are subsequently grouped in blocks, each of size N. The DFT of each block is to be computed in real time using the radix-2 decimation-in-frequency FFT algorithm. If the processor performs all operations sequentially, and takes 20 ps for computing each complex multiplication (including multiplications by 1 and -1) and the time required for addition/ subtraction is negligible, then the maximum value of N is _______.

[2016 : 2 Marks, Set-3]

27. Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to X[I] is shown in the figure. Let $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. In the figure, what should be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that X[I] is obtained correctly?



(a) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$

(b) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

(c) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$

(d) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$

[2019 : 2 Marks]

ANSWERS

1. (3·375)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (8)	**6.** (2·10 radians)	**7.** (3)	**8.** (c)	**9.** (a)
10. (0.0 to 0.0)	**11.** (b)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (11)
19. (6)	**20.** (c)	**21.** (c)	**22.** (7.9 to 8.1)		**23.** (7.9 to 8.1)	**24.** (112)	**25.** (d)	**26.** (8)
27. (d)								

EXPLANATIONS

1. $$x[n] = \left(\frac{2}{3}\right)^n u[n+3]$$

$$X(e^{j\Omega}) = \sum_{n=-3}^{\infty}\left(\frac{2}{3}\right)^n .e^{-j\Omega n} = \frac{\left(\frac{2}{3}\right)^{-3} .e^{-j3\Omega}}{1-\frac{2}{3}e^{-j\Omega}}$$

$$\Rightarrow \quad A = \left(\frac{3}{2}\right)^3 = \frac{27}{8} = 3.375$$

2. $$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$\therefore \quad y(n) = \left(\frac{1}{2}\right)^{2n} u^2(n)$$

$$y(n) = \left(\frac{1}{2}\right)^{2n} u(n) = \left(\left(\frac{1}{2}\right)^2\right)^n u(n)$$

$$\therefore \quad y(n) = \left(\frac{1}{4}\right)^n u(n)$$

$$\therefore \quad y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$y(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-iw}}$$

$$y(e^0) = \frac{4}{3}$$

3. From the known $y(n)$,

$$y(2n) = x(n-1)$$

$\therefore$ Fourier transform

$$= \frac{1}{2}e^{j\omega} + 1 + 2e^{-j\omega} + e^{-2j\omega} + \frac{1}{2}e^{-3j\omega}$$

$$= e^{-j\omega}\left[\frac{1}{2}\left(e^{2j\omega} + e^{-2j\omega}\right) + 2 + \left(e^{j\omega} + e^{-j\omega}\right)\right]$$

$$= e^{-j\omega}\left[\cos 2\omega + 2 + 2\cos\omega\right]$$

4. As, $x(n-n_0) \xleftrightarrow{\text{FT}} e^{-j\omega_0 n_0} \times \left(e^{j\omega}\right)$

then, $$Y(e^{j\omega}) = Ae^{-j\omega_0 n_0} \times \left(e^{j\omega}\right)$$

$$\Rightarrow \quad H(f\omega) = \frac{Y\left(e^{\varphi\omega}\right)}{X\left(e^{j\omega}\right)} = Ae^{-j\omega_0 n_0} \quad \text{...(i)}$$

Given, frequency reforms

$$H(e^{j\omega}) = \left|H\left(e^{j\omega}\right)\right| e^{j<H\left(e^{j\omega}\right)} \quad \text{...(ii)}$$

Comparing equation. (i) with (ii),

$$A = |H(e^{j\omega})|$$

and $$\angle H(e^{j\omega}) = -\omega_0 n_0$$

For LTI system, phase and frequency reform are periodic with 2π. The general form of $\angle H(e^{j\omega})$ is $-\omega_0 n_0 + 2\pi k$.

5. Plancheral's relation is given by

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) Y\left(e^{j\omega}\right) d\omega = \sum_{n=-\infty}^{\infty} x(n)y(n)$$

$$Y(e^{j\omega}) = \sin^2(2\omega) = \frac{1-\cos(4\omega)}{2}$$

$$= \frac{1}{2} - \frac{1}{4}e^{j4\omega} - \frac{1}{4}e^{-j4\omega}$$

$$y(n) = \frac{1}{2}\delta(n) - \frac{1}{4}\delta(n+4) - \frac{1}{4}\delta(n-4)$$

$$y(n) = \left\{-\frac{1}{4}, 0, 0, 0, \underset{\uparrow}{\frac{1}{2}}, 0, 0, 0, -\frac{1}{4}\right\}$$

$$x(n) = \left\{6, 3, \underset{\uparrow}{8}, 7, 4\right\}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) \cdot Y\left(e^{j\omega}\right) d\omega = 2\sum_{n=-\infty}^{\infty} x(n)y(n)$$

$$= 2\sum_{n=-\infty}^{\infty} x(n)y(n) = 2 \times 8 \times \frac{1}{2} = 8$$

6. Since $$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$

$$\therefore \quad H(e^{j\omega}) = \frac{1}{3}e^{j\omega}[1 + 2\cos\omega]$$

$$\therefore \quad H(e^{j\omega 0}) = 0,$$

$$\therefore \quad (1 + 2\cos\omega_0) = 0$$

$$\therefore \quad \cos\omega_0 = -\frac{1}{2}$$

$$\therefore \quad \omega_0 = \frac{2\pi}{3} = 2.10 \text{ radians}$$

7. X(k)= {1, 2, 3, 4, 5, 6, 7, 8}

$$\sum_{n=0}^{3} x[2n] = x[0] + x[2] + x[4] + x[6]$$

$$= 4.5 - 0.5 - 0.5j - 0.5 - 0.5 + 0.5j$$

$$= 4.5 - 1.5 = 3$$

8. The output of a discrete time system
$y(n) = \max |x(k)|; \quad -\infty \le k \le n$
The unit impulse response is
Now, apply $x(n) = \delta(n) \Rightarrow X(k) = \delta(k)$
So, $y(n) = \max|\delta(k)| = 1, n \ge 0 = u(n)$

9. As given that,

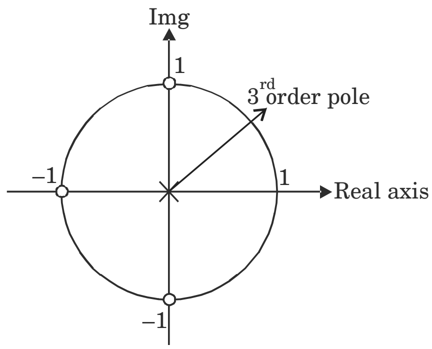
$$y(n) = \sum_{k=0}^{3} (-1)^k x(n-k)$$

$$y(n) = x(n) - x(n-1) + x(n-2) - x(n-3)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-1} + z^{-2} + z^{-3}$$

$$= \frac{z^3 - z^2 + z - 1}{z^3} = \frac{(z-1)(z^2+1)}{z^3}$$

Pole zero plot:



one zero at z = 1 & 2 zeros at $z = \pm j$
3 poles at z = 0
Hence, option (c) is correct.

10. As given that
For unit step input u(n), the output is $2\delta(n+1) + \delta(n) + \delta(n-1)$. We need to obtain the system response for input $\left(\frac{1}{2}\right)^n u(n)$.

$$s(n) = 2\delta(n+1) + \delta(n) + \delta(n-1)$$

[s(n) : step response]

$$h(n) = s(n) - s(n-1)$$

(h(n) : impulse response)

$$= 2\delta(n+1) + \delta(n) + \delta(n-1)$$
$$-[2\delta[(n-1)+1] + \delta(n-1) + \delta(n-1)-1]$$
$$= 2\delta(n+1) + \delta(n) + \delta(n-1) - 2\delta(n)$$
$$- \delta(n-1) - \delta(n-2)$$
$$= 2\delta(n+1) - \delta(n) - \delta(n-2)$$

If the input is $x(n) = \left(\frac{1}{2}\right)^n u(n)$ then its response is: $y(n) = x(n) * h(n)$

$$= \left[\left(\frac{1}{2}\right)^n u(n)\right] * [2\delta(n+1) - \delta(n) - \delta(n-2)]$$

$$= \left[\left(\frac{1}{2}\right)^n u(n) * 2\delta(n+1)\right] - \left[\left(\frac{1}{2}\right)^n u(n) * \delta(n)\right]$$
$$- \left[\left(\frac{1}{2}\right)^n u(n) * \delta(n-2)\right]$$

$$y(n) = 2\left(\frac{1}{2}\right)^{n+1} u(n+1) - \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^{n-2} u(n-2)$$

As we want Y(0), it is due to first 3 terms of y(n)

$$\text{So, } y(0) = 2\left(\frac{1}{2}\right)^1 u(1) - \left(\frac{1}{2}\right)^0 u(0) - \left(\frac{1}{2}\right)^2 u(-2)$$

$$= \left(2 \times \frac{1}{2} \times 1\right) - (1 \times 1) - \left[\left(\frac{1}{2}\right)^{-2} \times 0\right]$$

$$= 1 - 1 - 0 = 0$$

11. For discrete time Fourier transform (DTFT)

$$x(n) = \sum_{k=n} \frac{1}{n} X(e^{(jk\omega_0)})\, e^{jk\omega_o n}$$

When $\lim N \to \infty$,

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Putting $n = 0$,

$$x[0] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega \times 0}\, d\omega$$

$$\therefore \quad \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 2\pi\, x[0] = 2\pi \times 5 = 10\pi$$

12. Given: $\quad y(n) = \frac{1}{N} \sum_{r=0}^{N-1} x(r)\, x(n+r)$

It is auto correlation
Hence $\quad y(n) = r_{xx}(n)$

$$\therefore \quad r_{xx}(n) \xrightarrow[N]{DFT} |X(k)|^2$$

Alternately

$$X[n-r] = \sum_{n=0}^{N-1} x[n].W_N(N-r)n$$

$$\sum_{n=0}^{N-1} x(n).e^{-1}\left(\frac{2\pi}{N}\right)(N-r)n$$

Now, $e^{-1}\left(\frac{2\pi}{N}\right)(N-r) = e^{-12\pi n} e^{1}\left(\frac{2\pi}{N}\right)^{rn} = e^{1}\left(\frac{2\pi}{N}\right)^{rn}$

If x[n] is real, then x[n] = x[n]

and $x[N-r] = \sum_{n=0}^{N-1} x[n].e^{j}\left(\frac{2\pi}{N}\right)^{rn}$

$$= \left[\sum_{n=0}^{N-1} x[n].e^{-j}\left(\frac{2\pi}{N}\right)^{rn}\right] = X(k)$$

Hence, $\qquad Y[k] = [x(k)\{x^*[k]\} = |X(k)|^2$

13. 4 – point DFT of sequence [1, 0, 2, 3] is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & + & 2 & + & 3 \\ 1 & - & 2 & + & j3 \\ 1 & + & 2 & - & 3 \\ 1 & - & 2 & - & j3 \end{bmatrix} = \begin{bmatrix} 6 \\ -1+j3 \\ 0 \\ -1-j3 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} a+b+c+ \\ a+bW_3^{-1}+cW_3^{-2} \\ a+bW_3^{-2}+cW_3-1 \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & W_3^{2} & W_3^{1} \\ 1 & W_3^{1}W_3^{2} & W_3^{2}W_3^{1} \\ 1 & W_3^{2}W_3^{2} & W_3^{1}W_3^{1} \end{bmatrix}$$

$$\begin{bmatrix} \frac{(a+b+c)}{3} \\ \frac{\left(a+bW_3^{-1}+cW_3^{2}\right)}{3} \\ \frac{\left(a+bW_3^{-2}+cW_3^{-1}\right)}{3} \end{bmatrix}$$

where, $W_3^1 = e^{\frac{j2\pi}{3}}, W_3^2 = e^{\frac{j4\pi}{3}},$

$W_3^1 = e^{\frac{-j2\pi}{3}}, W_3^{-2} = e^{\frac{-j4\pi}{3}},$

$W_3^{-4} =, W_3^{-1} = e^{\frac{-j2\pi}{3}}.$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c \\ a \\ b \end{bmatrix}$$

14. $\qquad f(t) = e^{-t}u(t)$

$$F(s) = L\,[f(t)] = L\,[e^{-t}u(t)] = \frac{1}{s+1}$$

In frequency domain, $s = j\omega$

$$F(j\omega) = \frac{1}{(1+j\omega)} = \frac{-1}{\sqrt{1+\omega^2}} \angle \tan^{-1}\omega$$

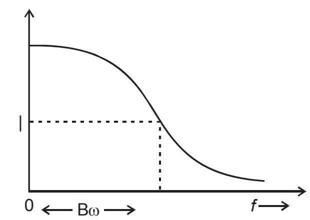
For 3 dB bandwidth,

$$\frac{1}{\sqrt{1+\omega^2}} = \frac{1}{\sqrt{2}}$$

$\therefore \quad \omega = +1$

$\omega = 2\pi f_c$

$\Rightarrow \; f_c = \frac{1}{2\pi}$ Hz



15. DFT of vector [a b c] is $[\propto \beta\gamma\delta]$.

$$\begin{bmatrix} \propto \\ \gamma \\ \beta \\ \delta \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & 1 & -j \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \begin{bmatrix} a+b+c+d \\ a-jb-c+jd \\ a-b+c-d \\ a+jb-c-jd \end{bmatrix} \qquad ...(i)$$

Given: [p q r s] = [a b c d]

$$\begin{bmatrix} a & b & c & d \\ b & a & b & c \\ c & c & a & b \\ d & d & d & a \end{bmatrix} \qquad ...(ii)$$

= [a_2 + bd + $c_2$bd ab + ab + cd+cd 2ac+b_2+d_2 2ad + 2bc]

DFT of [p q r s] is given as

$$[p\,q\,r\,s]\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \left[(p+q+r+s)(p-jq-pr+js)(p-q+r-s)\;(p+jq-r-js)\right]$$

$$= \left[\propto^2\ \beta^2\gamma^2\delta^2\right].$$

$\Rightarrow$ p + q + r+ s = $(a^2 + c^2 + 2bd) + (2ab + 2cd)$

$+ (b^2 + d^2 + 2ac) + (2ab + 2bc$ ... from eq(ii)

$= (a + b + c + d)^2$... from eq(i)

= a

16. This can be solved by directly using option and satisfying the condition given in question

$$X = \text{DFT}(x)$$

$$D_{FT}(D_{FT}(x)) = \text{DFT}(X) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} X[n]e^{-j\frac{2\pi}{N}nk}$$

DFT y [1 2 3 4]

$$X = \frac{1}{\sqrt{4}}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & +j & -1 & -j\end{bmatrix}\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

$$= \frac{1}{\sqrt{4}}\begin{bmatrix}10\\ 2+2j\\ 2\\ -2-j2\end{bmatrix}$$

DFT of (x) will not result in [1 2 3 4]

Try with DFT of y [1 2 3 2]

$$X = \frac{1}{\sqrt{4}}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & +j & -1 & -j\end{bmatrix}\begin{bmatrix}1\\2\\3\\2\end{bmatrix}$$

$$= \frac{1}{\sqrt{4}}\begin{bmatrix}8\\-2\\0\\-2\end{bmatrix} = \begin{bmatrix}4\\-1\\0\\-1\end{bmatrix}$$

$$\text{DFT of}\begin{bmatrix}4\\-1\\0\\-1\end{bmatrix} = \frac{1}{\sqrt{4}}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & +j & -1 & -j\end{bmatrix}\begin{bmatrix}4\\-1\\0\\-1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}2\\4\\6\\4\end{bmatrix} = \begin{bmatrix}1\\2\\3\\2\end{bmatrix}$$

Same as x

Then 'b' is right option

17. (c)

18. $x_1[n] = \{1, 2, 3, 0\}$, $x_2[n] = \{1, 3, 2, 1\}$

$X_3(k) = X_1(k)\,X_2(k)$

Based on the properties of DFT,

$x_1[n] \otimes x_2[n] = X_1(k)\,X_2(k) = x_3[n]$

Circular convolution between two 4-point signals is as follows :

$$\begin{bmatrix}1 & 0 & 3 & 2\\ 2 & 1 & 0 & 3\\ 3 & 2 & 1 & 0\\ 0 & 3 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\3\\2\\1\end{bmatrix} = \begin{bmatrix}9\\8\\11\\14\end{bmatrix}$$

$\therefore\ x_3[2] = 11$

19. Discrete Fourier Transform (DFT) of the 4-point sequence is

$$x_1[n] = x\left[\frac{n}{3}\right]$$

$X_1[K] = \{12, 2j, 0, -2j, 12, 2j, 0, -2j, 12, 2j, 0, -2j\}$

$X_1[8] = 12;$ $\qquad X_1(11) = -2j$

$$\left|\frac{X_1[8]}{X_1[11]}\right| = \left|\frac{12}{-2j}\right| = 6$$

20. As given that

$x(t) = \cos(200\pi t)$

$$t = \frac{n}{400}$$

$$x(n) = \cos\left(\frac{200\pi n}{400}\right) \Rightarrow x(n) = \cos\left(\frac{\pi n}{2}\right)$$

$$(\because n = 0, 1, \ldots, 7)$$

$x(n) = \{1, 0, -1, 0, 1, 0, -1, 0\}$

When we look at x(n), it contains only even samples.

Let us consider $z(n) = \{1 -1\ 1 -1\} \xleftrightarrow{\text{DFT}} Z(k)$

$$Z(k) = W_n|_{(n=4)}Z(n) = \begin{bmatrix}Z(0)\\Z(1)\\Z(2)\\Z(3)\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & 1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\4\\0\end{bmatrix}$$

N = 4

We know zero interpolation in time domain corresponds to replication of DFT spectrum

$$x(n) = z\left(\frac{n}{2}\right) \xleftrightarrow{\text{DFT}} X(k) = \{Z(k), Z(k)\}$$

$$= \{0,0,4,0,0,0,4,0\}$$

(or)

$$\cos\left(\frac{2\pi}{N}k_0 n\right) \leftrightarrow \frac{N}{2}\left[\delta(k-k_0) + \delta(k+k_0)\right]$$

$$\cos\left(\frac{n\pi}{2}\right) = \cos\left(\frac{2\pi}{8}2n\right) \leftrightarrow \frac{8}{2}[\delta(k-2)+\delta(k+2)]$$

So, non zero samples are –2, 2. From periodicity property non zero samples are –2 + 8, 2 = 6, 2.

21. As given that

$y(n) = x(n) * h(n)$

where, * represents linear convolution

$Z(n) = \text{IDFT}\,[X(k).\ H(k)]$

16 point DFT is considered

We need to find for what value of k, $y(k) = z(k)$

$\text{IDFT}[X(k).\ H(k)] = x(n) \circledast h(n)$ [$\circledast$: circular convolution]

where,

x(n) have samples

h(n) have 16 samples

As we know that the length of linear convolution is $= N_1 + N_2 - 1$

Then x(n) * h(n) wil have 16 + 16 – 1 = 31 number of samples $x(n) \circledast h(n)$ will have 16 number of samples.

While computing circular convolution through liner convolution, we used to add last (k – 1) number of samples of linear convolution in beginning without disturbing the kth sample of linear and circular convolution. So the k^{th} samples of linear convolution and circular convolution same.

Now, take an example, $x[n] = \left[\underset{\uparrow}{1},2,3\right]$

$$h(n) = \left[\underset{\uparrow}{4},5,6\right]$$

x(n) * h(n)					
	1	2	3		
	4	5	6		
	4	8	12		
		5	10	15	
			6	12	18
	4	13	28	27	18
	27	28			
	31	41	28		

So, $y(n) = x(n) * h(n) = \left[\underset{\uparrow}{4},13,28,27,18\right]$

$z(n) = x(n) \circledast h(n) = [31, 31, 28]$

Here we have taken 3 samples in both x(n), h(n), and can see the 3^{rd} samples of y(n), z(n) are same i.e., 28. [3^{rd} sample is equivalent to n = 2 or k = 2]

Instead of 3 samples, if we take 16 samples then 16^{th} samples will appear same in both y(n), z(n) [16^{th} samples is equivalent to n = 15 or k = 15 sincestarting value of n/k is 0 not 1].

Hence, option (c) is correct answer.

22. As given that

$$x(n) = 2^{n-1}\,u(-n+2)$$
$$y(n) = 2^{-n+2}\,u(n+1)$$

$$I = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega = ?$$

Let, $\quad P(e^{j\omega}) = X(e^{j\omega})Y(e^{-j\omega})$

Then $\quad \frac{1}{2\pi}\int_0^{2\pi} P(e^{j\omega})d\omega = P(0) = P(n)|_{n=0}$

$$P(n) = \frac{1}{2\pi}\int_0^{2\pi} P(e^{j\omega})e^{-j\omega n}d\omega$$

If we make n = 0 in above equation then

$$P(0) = \frac{1}{2\pi}\int_0^{2\pi} P(e^{j\omega})d\omega$$

If $\quad P(e^{j\omega}) = X(e^{j\omega})Y(e^{-j\omega})$

Then $\quad P(n) = x(n) * y(-n) \qquad [\because\ y(-n) \leftrightarrow Y(e^{-j\omega})]$

$\quad P(n) = x(n) * g(n) \qquad$ [Where $g(n) = y(-n)$]

$$= \sum_{k=-\infty}^{\infty} x(k)g(n-k) \qquad [\because g(n) = 2^{n+2}u(-n+1)]$$

$$= \sum_{k=-\infty}^{\infty} \left[2^{k-1}u(-k+2)\right]\left[2^{n-k+2}u[-(n-k)+1]\right]$$

$$= \sum_{k=-\infty}^{\infty} (2^{k-1})(2^{n-k+2})u(-k+2)u(k-n+1)$$

$$\Rightarrow P(0) = \sum_{k=-\infty}^{\infty} 2^{k-1}.2^{-k+2}\left[u(-k+2).u(k+1)\right]$$

$$\left[\begin{array}{l} u(-k+2) = 1; -\infty \le k \le 2 \\ u(k+1) = 1;\ -1 \le k \le \infty \\ u(-k+2),\ u(k+1) = \begin{matrix} 1; & -1 \le k \le 2 \\ 0; & \text{else} \end{matrix} \end{array}\right]$$

$$\Rightarrow \quad P(0) = \sum_{k=-1}^{2} 2^{k-1+(-k+2)}$$

$$= \sum_{k=-1}^{2} 2$$

$$= \sum_{k=-1}^{2} 2(1)^k$$

$= 2(1)^{-1} + 2(1)^0 + 2(1)^1 + 2(1)^2$

$= (2 \times 1) + (2 \times 1) + (2 \times 1) + (2 \times 1)$

$= 2 + 2 + 2 + 2 = 8$

23. As given expression;

$$x[k] = \sum_{n=0}^{7} x[n] \exp\left[-j\frac{2\pi}{8} n\pi\right]$$

Here $j = \sqrt{-1}$ if

$\bar{x} = [1, 0, 0, 0, 2, 0, 0, 0]$

$$x(n) \xrightarrow{\text{DFT}} \xrightarrow{\text{DFT}} NX(-k)|_{\text{mod N}}$$

$$= NX(-k)|_{\text{mod N}} = Y(k) \quad \text{....(a)}$$

As we know that,

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k) e^{j\frac{2\pi}{N}kn}$$

By Putting n = 0, N = 8

$$y(0) = \frac{1}{8}\sum_{k=0}^{7} Y(k) \quad \text{....(b)}$$

$$\bar{y}(n) = N.X(-k)|_{\text{mod}\cdot N}$$

Now, from equation (a), $y(n) = 8X(-k)|_{\text{mod N}}$

$= 8X(N - k)$

$= 8[1, 0, 0, 0, 2, 0, 0, 0]$

$= [8, 0, 0, 0, 16, 0, 0, 0]$

from equation (b), $y(0) = 8$

24. As we know that

For, the sequence x(n) = {1, 2, 1, 3}

$$Y[k]_{K=0} = \sum_{n=0}^{3} y[n]$$

$$= \sum_{n=0}^{3} N^2 x[n] \qquad (\therefore y(n) = N^2 x[n])$$

$$= N^2\left[x[0] + x[1] + x[2] + x[3]\right] \qquad (\therefore N = 4)$$

$$= (4)^2[1 + 2 + 1 + 3]$$

$$= 16 \times 7 = 112$$

Hence, the sequence x(n), the value of y(0) is 112.

25. For N-point Fast Fourier Transform (FFT), with $N = 2^m$.

We need only one complex multiplication to compute a butterfly.

26. The number of complex multiplications required

for DIF-FFT $= \left(\frac{N}{2}\log_2 N\right)$

$$\therefore \left(\frac{N}{2}\log_2 N\right)(20\ \mu\text{ sec}) = 125\ \mu\text{sec}$$

27. Using DIT algorithm are can obtain FFT coefficient [X(I)].

The given butterfly structure is a standard structure where

$$a_1 = W_6^0 = 1$$

$$a_2 = W_6^1 = W_6$$

$$a_3 = W_6^2$$